Geometry of Locally Conformal C_{12} −Manifolds

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ABSTRACT

This study deals with the class of locally conformal C_{12} −manifolds in such a way that the characterization identity for the aforementioned class is produced. Furthermore, the first clan of Cartan's structure equations is determined when the components of Kirichenko's tensors on associated G-structures for locally conformal C_{12} −manifolds are determined. The second clan of Cartan's structural equations for locally conformal C_{12} −manifolds was also completed.

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1. Introduction

The classification of almost contact metric (ACR-) manifolds done in 1990 by Chinea and Gonzalez [11]. The authors also characterized the class C_{12}. This class investigated by more than one author. For instance, Bouzir et al. [10] studied the class C_{12}, when the dimension is 3. Abass and Al-Zamil [2] discussed the geometry of Weyl tensor on C_{12} −manifolds. Moreover, some extension of the aforementioned class established. As an example of this extension is the class $C_5 \oplus C_{12}$ that studied by de Candia and Falcitelli [13].

On the other hand, Olszak [19] discovered the locally conformal almost (LCA-) cosymplectic manifolds in 1989. Later, many authors deal with the class of LCA-cosymplectic manifolds such as Massamba and Mavambou [18] and Abood and Al-Hussaini [7, 8]. While, Kirichenko and Kharitonova [15] studied the normal class of LCA-cosymplectic manifolds.

In 1992, Chinea and Marrero [12], continued with the study of globally and locally conformal changes for ACR-structures furnished by some examples. In that way, Kirichenko and Uskorev [16] determined the conformal transformation of six Kirichenko's structure tensors. Recently, many classes of locally conformal structures instituted. For example, the class of conformal Kenmotsu that found and studied by Abdi and Abedi [5] and Abdi [3, 4].

The researchers can be obtained new classes of ACR-manifolds and new transformations of ACR-structures that created by Abood and Abass [6] and Beldjilali and Akyol [9] respectively. Furthermore, the researchers can be observed that in this paper after introduction, the preliminaries on ACR-manifolds in section 2. Next, in section 3, the locally conformal C_{12} −manifolds characterized and in the last section, Cartan's structure equations of locally conformal C_{12} −manifolds determined.

1. Preliminaries

Let M^n be a smooth n −topological manifold and let $X(M)$ be Lie algebra of smooth vector fields over M^n.

Remark 2.1. [17] The Lie algebra that mentioned above is defined as a real vector space $\mathcal{G}$ furnished with a map called the bracket from $\mathcal{G} \times \mathcal{G}$ to $\mathcal{G}$, usually denoted by $(X, Y) \longmapsto [X, Y]$, that satisfies the following properties for all $X, Y, Z \in \mathcal{G}$:

(i) Bilinearity: For $a\ , b \in \mathbb{R}$,

$$[aX + bY, Z] = a[X, Z] + b[Y, Z];$$

$$[X, aY + bZ] = a[X, Y] + b[X, Z].$$

(ii) Antisymmetry: $[X, Y] = -[Y, X]$.

(iii) Jacobi identity: $\big[X, [Y, Z]\big] + \big[Z, [X, Y]\big] + \big[Y, [Z, X]\big] = 0$.

Definition 2.1. [1] An ACR- manifold is a Riemannian manifold $(M^{2n+1}, g = \langle .,. \rangle)$ with structure (η, ξ, Φ) consist of 1-form η, a vector field ξ and a tensor field Φ of type $(1\ ,\ 1)$ satisfies the following:

1) $\eta(\xi) = 1$;
2) $\Phi(\xi) = 0$;
3) $\eta \circ \Phi = 0$;
4) $\Phi^2 = -id + \eta \otimes \xi$;
5) $\langle \Phi X,\ \Phi Y \rangle = \langle X, Y \rangle - \eta(X)\eta(Y)$; $\forall\, X,\ Y\ \in X(M)$.

Notation. The symbol $\Lambda(M)$ is Grassmann algebra and equal to $\bigoplus_{r=0}^{\infty} \Lambda_r(M)$, where $\Lambda_r(M)$ is the algebra of differential r −forms over M.

Theorem 2.1. [17] If M^n is a smooth n −topological manifold then there exists a unique operator $d : \Lambda(M) \longrightarrow \Lambda(M)$ satisfies the following properties:

1. d is linear over $\mathbb{R}$.
2. $d\big(\Lambda_k(M)\big) \subset \Lambda_{k+1}(M)$.
3. $d(\omega_1 \wedge \omega_2) = d\omega_1 \wedge \omega_2 + (-1)^k\, \omega_1 \wedge d\omega_2$; where $\omega_1 \in \Lambda_k(M)$; $\omega_2 \in \Lambda_l(M)$.
4. $d^2 = d \circ d = 0$.
5. $df(X) = X(f)$; $\forall\, f \in C^\infty(M)$ & $\forall\, X \in X(M)$.

Now, the associated G-structure method that mentioned in [14] can be introduced here and it summarized as follows:

Suppose that $\{\xi, e_1, e_2, \dots, e_n, e_{\hat{1}}, e_{\hat{2}}, \dots, e_{\hat{n}}\}$ is an orthonormal basis of vector fields over an ACR-manifolds $(M^{2n+1}, \xi, \eta, \Phi, g)$, where $\hat{a} = a + n$; $a = 1, 2, \dots, n$. So, a new basis $\{\xi, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n, \varepsilon_{\hat{1}}, \varepsilon_{\hat{2}}, \dots, \varepsilon_{\hat{n}}\}$ introduced, such that $\varepsilon_a = \frac{1}{\sqrt{2}}\big(e_a - \sqrt{-1}\,\Phi e_a\big)$; $\varepsilon_{\hat{a}} = \frac{1}{\sqrt{2}}\big(e_a + \sqrt{-1}\,\Phi e_a\big)$. Therefore, on associated G-structure the tensors g and Φ are written as follow respectively [20]:

$$g=\begin{pmatrix}1&0&0\\0&0&I_n\\0&I_n&0\end{pmatrix}\quad;\qquad \Phi=\begin{pmatrix}0&0&0\\0&\sqrt{-1}\,I_n&0\\0&0&-\sqrt{-1}\,I_n\end{pmatrix},$$

where I_n is the identity matrix of rank n.

On the other side, the six structure tensors on ACR-manifold $(M^{2n+1},\ \xi,\ \eta,\Phi,\ g)$ which are determined in Abood and Abass [6] as follow:

$$B(X,Y)=-\tfrac{1}{8}\{\Phi\circ\nabla_{\Phi^2Y}(\Phi)(\Phi^2X)+\Phi\circ\nabla_{\Phi Y}(\Phi)(\Phi X)+\Phi^2\circ\nabla_{\Phi Y}(\Phi)(\Phi^2X)$$
$$-\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)(\Phi X)\};$$

$$C(X,Y)=-\tfrac{1}{8}\{-\Phi\circ\nabla_{\Phi^2Y}(\Phi)(\Phi^2X)+\Phi\circ\nabla_{\Phi Y}(\Phi)(\Phi X)+\Phi^2\circ\nabla_{\Phi Y}(\Phi)(\Phi^2X)$$
$$+\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)(\Phi X)\};$$

$$D(X)=\tfrac{1}{4}\{2\Phi\circ\nabla_{\Phi^2X}(\Phi)\xi-2\Phi^2\circ\nabla_{\Phi X}(\Phi)\xi-\Phi\circ\nabla_{\xi}(\Phi)(\Phi^2X)+\Phi^2\circ\nabla_{\xi}(\Phi)(\Phi X)\};$$

$$E(X)=-\tfrac{1}{2}\{\Phi\circ\nabla_{\Phi^2X}(\Phi)\xi+\Phi^2\circ\nabla_{\Phi X}(\Phi)\xi\};$$

$$F(X)=\tfrac{1}{2}\{\Phi\circ\nabla_{\Phi^2X}(\Phi)\xi-\Phi^2\circ\nabla_{\Phi X}(\Phi)\xi\};$$

$$G=\Phi\circ\nabla_{\xi}(\Phi)\xi.$$

The above tensors have components on associated G-structure given in Kirichenko and Dondukova [14] respectively as follow:

1. $B=\{B^i_{jk}\}$; $B^a_{\hat{b}c}=B^{ab}{}_c=-\frac{\sqrt{-1}}{2}\Phi^a_{\hat{b},c}$; $B^{\hat{a}}_{b\hat{c}}=B_{ab}{}^c=\frac{\sqrt{-1}}{2}\Phi^{\hat{a}}_{b,\hat{c}}$;
2. $C=\{C^i_{jk}\}$; $C^a_{\hat{b}\hat{c}}=C^{abc}=\frac{\sqrt{-1}}{2}\Phi^a_{\hat{b},\hat{c}}$; $C^{\hat{a}}_{bc}=C_{abc}=-\frac{\sqrt{-1}}{2}\Phi^{\hat{a}}_{b,c}$. So,
   $$B^{abc}=C^{a[bc]}\ ;\quad B_{abc}=C_{a[bc]}\ ;$$
3. $D=\{D^i_j\}$; $D^a_{\hat{b}}=B^{ab}=\sqrt{-1}\left(\Phi^a_{0,\hat{b}}-\frac{1}{2}\Phi^a_{\hat{b},0}\right)$;
   $$D^{\hat{a}}_b=B_{ab}=-\sqrt{-1}\left(\Phi^{\hat{a}}_{0,b}-\tfrac{1}{2}\Phi^{\hat{a}}_{b,0}\right)\ ;$$
4. $E=\{E^i_j\}$; $E^a_b=B^a{}_b=\sqrt{-1}\,\Phi^a_{0,b}$; $E^{\hat{a}}_{\hat{b}}=B_a{}^b=-\sqrt{-1}\,\Phi^{\hat{a}}_{0,\hat{b}}$;
5. $F=\{F^i_j\}$; $F^a_{\hat{b}}=F^{ab}=\sqrt{-1}\,\Phi^0_{\hat{a},\hat{b}}$; $F^{\hat{a}}_b=F_{ab}=-\sqrt{-1}\,\Phi^0_{a,b}$. So,
   $$C^{ab}=F^{[ab]}\ ;\quad C_{ab}=F_{[ab]}\,;$$
6. $G=\{G^i\}$; $G^a=C^a=-\sqrt{-1}\,\Phi^0_{\hat{a},0}$; $G_a=C_a=\sqrt{-1}\,\Phi^0_{a,0}$.

Suppose that $\{\omega^0 = \omega\ ,\ \omega^1\ ,\ \ldots,\ \omega^n, \omega_1\ , \ldots, \omega_n\}$ is a set of 1 – forms which are dual of the set $\{\ \xi,\ \varepsilon_1, \varepsilon_2, \ldots\ , \varepsilon_n\ , \varepsilon_{\hat{1}}\ , \varepsilon_{\hat{2}}\ , \ldots, \varepsilon_{\hat{n}}\ \}$ respectively. Then next theorem is achieved.

Theorem 2.2. [14] On associated G-structure, the first clan of Cartan's structure equations for an ACR-manifold $(M^{2n+1},\ \xi, \eta, \Phi, g)$ are given by

1) $d\omega^a = -\theta^a_b \wedge \omega^b + B^{ab}{}_c\, \omega^c \wedge \omega_b + B^{abc}\omega_b \wedge \omega_c + B^a{}_b \omega \wedge \omega^b + B^{ab}\omega \wedge \omega_b$;
2) $d\omega_a = \theta^b_a \wedge \omega_b + B_{ab}{}^c\, \omega_c \wedge \omega^b + B_{abc}\, \omega^b \wedge \omega^c + B_a{}^b \omega \wedge \omega_b + B_{ab}\, \omega \wedge \omega^b$;
3) $d\omega = C_{bc}\, \omega^b \wedge \omega^c + C^{bc}\, \omega_b \wedge \omega_c + C^b_c\, \omega^c \wedge \omega_b + C_b\ \omega \wedge \omega^b + C^b\ \omega \wedge \omega_b$,

where θ^a_b are components of Riemannian connection ∇ and $C^b_c = B^b{}_c - B_c{}^b$.

Definition 2.2. [16] Let $(M^{2n+1},\ \xi, \eta, \Phi, g)$ be an ACR-manifold. A conformal change of the ACR-structure on M^{2n+1} is a change of the form:

$$\tilde{\Phi} = \Phi\ , \quad \tilde{\xi} = e^{\alpha}\, \xi \quad , \quad \tilde{\eta} = e^{-\alpha}\, \eta \quad , \quad \tilde{g} = e^{-2\alpha}\, g$$

where α is a smooth function on M. So, $(\tilde{\Phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is called a locally conformal ACR-structure related to ACR-manifold $(M^{2n+1},\ \xi, \eta, \Phi, g)$.

2. The locally conformal C_{12} −structures

In this section, we discussed the ACR-manifold $(M^{2n+1},\ \xi, \eta, \Phi, g)$ that its locally conformal ACR-manifold $(M^{2n+1}, \tilde{\Phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is of class C_{12} .

Theorem 3.1. If $(M^{2n+1},\ \xi, \eta, \Phi, g)$ is a locally conformal C_{12} −manifold, then it satisfies the following identity:

$$\nabla_X(\Phi)Y = -g(X\ , \Phi Y)\ \alpha^{\#} + g(X, Y)\Phi(\alpha^{\#}) + d\alpha(\Phi Y)X - d\alpha(Y)\Phi X$$
$$-\eta(X)\{\ \eta(Y)\ [\Phi(\nabla_\xi\ \xi) + \Phi(\alpha^{\#})]\ + g(\nabla_\xi\ \xi\ , \Phi Y)\xi + g(\alpha^{\#}\ , \Phi Y)\xi\ \}.$$

Proof. Suppose that $(M^{2n+1},\ \xi, \eta, \Phi, g)$ is a locally conformal C_{12} −manifold with Levi– Civita connection ∇. Then $(M^{2n+1}, \tilde{\xi}, \tilde{\eta}, \Phi, \tilde{g})$ is a C_{12} −manifold with Levi – Civita connection $\tilde{\nabla}$, where

$$\tilde{\xi} = e^{\alpha}\xi \quad , \quad \tilde{\eta} = e^{-\alpha}\eta \quad , \ \tilde{g} = e^{-2\alpha} g.$$

From Chinea and Gonzalez [11], a C_{12} −manifold $(M^{2n+1}, \tilde{\xi}, \tilde{\eta}, \Phi, \tilde{g})$ satisfies:

$$\tilde{\nabla}_X(\Phi)Y = -\tilde{\eta}(X)\{\tilde{\eta}(Y)\,\Phi(\tilde{\nabla}_{\tilde{\xi}}\,\tilde{\xi}) + \tilde{g}(\tilde{\nabla}_{\tilde{\xi}}\,\tilde{\xi}\,,\Phi Y)\tilde{\xi}\,\}; \quad (3.1)$$

for all $X, Y \in X(M)$. According to Kirichenko and Uskorev [16], ∇ and $\tilde{\nabla}$ are related by:

$$\tilde{\nabla}_X(\Phi)Y = \nabla_X(\Phi)Y + g(X\,,\Phi Y)\,\alpha^{\#} - g(X,Y)\Phi(\alpha^{\#}) - d\alpha(\Phi Y)X$$
$$+ d\alpha(Y)\Phi X\,; \quad (3.2)$$

for all $X, Y \in X(M)$, where $\alpha^{\#} = \mathrm{grad}(\alpha)$. Then the substituting of equation (3.2) in (3.1), produce the following:

$$\nabla_X(\Phi)Y = -g(X\,,\Phi Y)\,\alpha^{\#} + g(X,Y)\Phi(\alpha^{\#}) + d\alpha(\Phi Y)X - d\alpha(Y)\Phi X$$
$$-e^{-\alpha}\eta(X)\{\,e^{-\alpha}\eta(Y)\Phi\big(\nabla_{\tilde{\xi}}\,\tilde{\xi} - 2d\alpha(\tilde{\xi})\tilde{\xi} + g(\tilde{\xi}\,,\tilde{\xi})\alpha^{\#}\big)$$
$$+\,e^{-2\alpha}g\big(\nabla_{\tilde{\xi}}\,\tilde{\xi} - 2d\alpha(\tilde{\xi})\tilde{\xi} + g(\tilde{\xi}\,,\tilde{\xi})\alpha^{\#}\,,\Phi Y\big)e^{\alpha}\xi\,\}\,;$$
$$= -g(X\,,\Phi Y)\,\alpha^{\#} + g(X,Y)\Phi(\alpha^{\#}) + d\alpha(\Phi Y)X - d\alpha(Y)\Phi X$$
$$-e^{-\alpha}\eta(X)\{\,e^{-\alpha}\eta(Y)\big[\Phi\big(\nabla_{e^{\alpha}\xi}(e^{\alpha}\xi\,)\big) + e^{2\alpha}\Phi(\alpha^{\#})\big]$$
$$+e^{-\alpha}\big[g\big(\nabla_{e^{\alpha}\xi}(e^{\alpha}\xi)\,,\Phi Y\big) + e^{2\alpha}g(\alpha^{\#}\,,\Phi Y)\big]\xi\,\}\,.$$

Noted that $\nabla_{e^{\alpha}\xi}(e^{\alpha}\xi) = e^{\alpha}\nabla_{\xi}(e^{\alpha}\xi) = e^{\alpha}\{e^{\alpha}\nabla_{\xi}\xi + \xi(e^{\alpha})\xi\}$. Thus,

$$\nabla_X(\Phi)Y = -g(X\,,\Phi Y)\,\alpha^{\#} + g(X,Y)\Phi(\alpha^{\#}) + d\alpha(\Phi Y)X - d\alpha(Y)\Phi X$$
$$-e^{-\alpha}\eta(X)\{\,e^{-\alpha}\eta(Y)\big[e^{2\alpha}\Phi\big(\nabla_{\xi}\xi\big) + e^{2\alpha}\Phi(\alpha^{\#})\big]$$
$$+e^{-\alpha}\big[e^{2\alpha}g\big(\nabla_{\xi}\xi\,,\Phi Y\big) + e^{2\alpha}g(\alpha^{\#}\,,\Phi Y)\big]\xi\,\}\,;$$
$$= -g(X\,,\Phi Y)\,\alpha^{\#} + g(X,Y)\Phi(\alpha^{\#}) + d\alpha(\Phi Y)X - d\alpha(Y)\Phi X$$
$$-\eta(X)\{\eta(Y)\,\big[\Phi\big(\nabla_{\xi}\,\xi\big) + \Phi(\alpha^{\#})\big] + g\big(\nabla_{\xi}\,\xi\,,\ \Phi Y\big)\xi + g(\alpha^{\#}\,,\Phi Y)\xi\,\}. \qquad \blacksquare$$

Theorem 3.2. If $(M^{2n+1}, \xi, \eta, \Phi, g)$ is a locally conformal C_{12} −manifold, then Kirichenko's tensors on M are given by:

(1) $B(X,Y) = \frac{1}{2}\{\,g(\Phi^2Y\,,X)\Phi^2(\alpha^{\#}) - g(\Phi Y\,,X)\Phi(\alpha^{\#}) - d\alpha(\Phi^2X)\Phi^2Y$

$-d\alpha(\Phi X)\Phi Y\,\}$;

(2) $C = D = F = 0$; and

(3) $E(X) = -d\alpha(\xi)(\Phi^2 X)$.

Proof. Suppose that $(M^{2n+1}, \xi, \eta, \Phi, g)$ is a locally conformal C_{12} −manifold with Kirichenko's tensors $B, C, \dots, G$, and let $(M^{2n+1}, \tilde{\xi}, \tilde{\eta}, \Phi, \tilde{g})$ be its C_{12} −manifold with Kirichenko's tensors $\widetilde{B}, \widetilde{C}, \dots, \widetilde{G}$. So, from Kirichenko and Uskorev [16], we have the following identities for all $X, Y \in X(M)$:

$$\widetilde{B}(X,Y) = B(X,Y) - \frac{1}{2}\{\, g(\Phi^2 Y, X)\Phi^2(\alpha^\#) - g(\Phi Y, X)\Phi(\alpha^\#) - d\alpha(\Phi^2 X)(\Phi^2 Y)$$

$$-d\alpha(\Phi X)(\Phi Y)\,\};$$

$$\widetilde{C}(X,Y) = C(X,Y); \quad \widetilde{D}(X) = e^{\alpha} D(X); \quad \widetilde{F}(X) = e^{\alpha} F(X);$$

$$\widetilde{E}(X) = e^{\alpha}\big(E(X) + d\alpha(\xi)(\Phi^2 X)\big); \quad \widetilde{G} = e^{2\alpha}\Big(G - \Phi^2(\alpha^\#)\Big).$$

On the other hand, from [2] deduced that $\widetilde{B} = \widetilde{C} = \widetilde{D} = \widetilde{E} = \widetilde{F} = 0$. Then the results attain. ∎

Corollary 3.1. If $(M^{2n+1}, \xi, \eta, \Phi, g)$ is a locally conformal C_{12} −manifold, then Kirichenko's tensors on M have the following components on associated G-structure:

(1) $B_{ab}{}^{c} = 2\alpha_{[a}\,\delta^{c}_{b]}$, $B^{ab}{}_{c} = 2\alpha^{[a}\,\delta^{b]}_{c}$;

(2) $B^{abc} = B^{ab} = C^{ab} = 0$; $B_{abc} = B_{ab} = C_{ab} = 0$;

(3) $B^{a}_{\ b} = \alpha_o\,\delta^{a}_{b}$, $B_{a}{}^{b} = \alpha_o\,\delta^{b}_{a}$,

where α_i are the components of 1-form $d\alpha$ on associated G-structure with $i = 0, a, \hat{a}$.

Proof. Based on Theorem 3.2, we have the components of tensors C, D, F on associated G-structure are equal to zero. So, item (2) of this corollary achieved. Also, from Theorem 3.2; item (1), we get:

$$B(X,Y) = \tfrac{1}{2}\{\, g(\Phi^2 Y, X)\Phi^2(\alpha^\#) - g(\Phi Y, X)\Phi(\alpha^\#) - d\alpha(\Phi^2 X)\Phi^2 Y$$

$$-d\alpha(\Phi X)\Phi Y\,\};$$

$= \frac{1}{2}\{ [-g(Y, X) + \eta(X)\eta(Y)][-\alpha^{\#} + \eta(\alpha^{\#})\xi] - g(\Phi Y, X)\Phi(\alpha^{\#})$

$-\{d\alpha(-X + \eta(X)\xi)(-Y + \eta(Y)\xi) - d\alpha(\Phi X)\Phi Y\}$.

Therefore, the components of B on associated G-structure are given by:

$$B^i_{jk}X^jY^k\varepsilon_i = \frac{1}{2}\{ \left(-g_{jk}X^jY^k + \eta_j\eta_kX^jY^k\right)\left(-\alpha^i + \eta_l\alpha^l\delta^i_0\right)\varepsilon_i - \Omega_{jk}X^jY^k\Phi^i_l\alpha^l\varepsilon_i$$

$$-\left(-g_{lj}\alpha^lX^j + \eta_jX^jg_{0l}\alpha^l\right)\left(-\delta^i_kY^k\varepsilon_i + \eta_kY^k\ \delta^i_0\varepsilon_i\right)$$

$$-\Omega_{lj}\alpha^lX^j\Phi^i_kY^k\varepsilon_i\},$$

where $\Omega(X,Y) = g(X, \Phi Y)$. Then

$$B^i_{jk} = \frac{1}{2}\{ g_{jk}\alpha^i - g_{jk}\eta_l\alpha^l\delta^i_0 - \eta_j\eta_k\alpha^i + \eta_j\eta_k\eta_l\alpha^l\delta^i_0 - \Omega_{jk}\Phi^i_l\alpha^l - g_{lj}\delta^i_k\alpha^l$$

$$+g_{lj}\alpha^l\eta_k\delta^i_0 + \eta_jg_{0l}\alpha^l\delta^i_k - \eta_jg_{0l}\alpha^l\eta_k\delta^i_0 - \Omega_{lj}\alpha^l\Phi^i_k\}. \quad (3.3)$$

Now, we put $i = a$; $j = \hat{b}$; $k = c$ in equation (3.3), we get:

$$B^{ab}{}_c = \frac{1}{2}\{ \delta^b_c\alpha^a - \sqrt{-1}\delta^b_c\sqrt{-1}\,\delta^a_d\alpha^d - \delta^b_d\alpha^d\delta^a_c + \delta^b_d\alpha^d\sqrt{-1} * \sqrt{-1}\delta^a_c\} = 2\alpha^{[a}\delta^{b]}_c .$$

So, $B_{ab}{}^c = \overline{B^{ab}{}_c} = 2\alpha_{[a}\delta^c_{b]}$.

Again from Theorem 3.2; item (3), we have $E(X) = -d\alpha(\xi)(\Phi^2X)$. Then the components of E on associated G-structure are given by:

$$E^i_j = -\alpha_0\left(-\delta^i_j + \eta_j\xi^i\right). \quad (3.4)$$

If we put $i = a$; $j = b$ in equation (3.4), then we get:

$B^a{}_b = \alpha_0\delta^a_b$ and $B_a{}^b = \overline{B^a{}_b} = \alpha_0\delta^b_a$. This implies that

$$C^b_c = B^b{}_c - B_c{}^b = \alpha_0\delta^b_c - \alpha_0\delta^b_c = 0. \quad ■$$

Theorem 3.3. The first clan of Cartan's structure equations of locally conformal $C_{12} -$ manifold on associated G-structure are given by:

1) $d\omega = C_b \omega \wedge \omega^b + C^b \omega \wedge \omega_b$;

2) $d\omega^a = -\theta_b^a \wedge \omega^b + 2\alpha^{[a} \delta_c^{b]} \omega^c \wedge \omega_b + \alpha_o \delta_b^a \omega \wedge \omega^b$;

3) $d\omega^{\hat{a}} = d\omega_a = \theta_a^b \wedge \omega_b + 2\alpha_{[a}\delta_{b]}^c \omega_c \wedge \omega^b + \alpha_o \delta_a^b \omega \wedge \omega_b$.

Proof. According to Theorem 2.2 and Corollary 3.1, yield the results. ■

4. Main Theorem

In this section, we investigate the second clan of Cartan's structure equations of locally conformal $C_{12} -$ manifold.

Theorem 4.1. The second clan Cartan's structure equations of locally conformal $C_{12} -$ manifold on G-structure are given by:

(1) $dC^b + C^d \theta_d^b = C^{bd} \omega_d + C^b{}_d \omega^d + C^{bo} \omega$;

(2) $dC_b - C_d \theta_b^d = C_{bd} \omega^d + C_b{}^d \omega_d + C_{bo} \omega$;

(3) $d\theta_b^a = -\theta_b^a \wedge \theta_c^b + A_{bch}^{ad} \theta_d^c \wedge \omega^h + A_{bc}^{adh} \theta_d^c \wedge \omega_h + A_{bcd}^a \omega^c \wedge \omega^d$
$+A_{bc}^{ad} \omega^c \wedge \omega_d + A_{bc0}^a \omega^c \wedge \omega + A_b^{acd} \omega_c \wedge \omega_d + A_b^{ac0} \omega_c \wedge \omega$;

(4) $d\alpha^a = -\alpha^c \theta_c^a + \alpha_d^{ac} \theta_c^d + \alpha_c^a \omega^c + \alpha^{ac} \omega_c + \alpha^{ao} \omega$;

(5) $d\alpha_a = \alpha_c \theta_a^c + \alpha_{ac}{}^d \theta_d^c + \alpha_a{}^c \omega_c + \alpha_{ac} \omega^c + \alpha_{a0} \omega$;

(6) $d\alpha_0 = \alpha_{0c} \omega^c + \alpha_0^c \omega_c + \alpha_{00} \omega$,

where $C^{[bd]} = C_{[bd]} = 0$; $C_b{}^d - C_h \alpha^h \delta_b^d + C_b \alpha^d - C^d{}_b + C^h \alpha_h \delta_b^d - C^d \alpha_b = 0$;

$A_{[b|c|h]}^{ad} = 0$; $A_{[bcd]}^a = 0$; $-A_{bc}^{adh} + \alpha_c^{hd} \delta_b^a - \alpha_c^{ad} \delta_b^h = 0$;

$-A_{[bc]}^{ad} - \frac{1}{2}\alpha^d B_{bc}{}^a + \alpha_{[c}^d \delta_{b]}^a - \alpha_{[c}^a \delta_{b]}^d + \alpha^a B_{bc}{}^d - \alpha^h \alpha_h \delta_{[b}^a \delta_{c]}^d = 0$;

$-A_{[bc]0}^a + \alpha_{0[c} \delta_{b]}^a + \alpha_o C_{[b} \delta_{c]}^a = 0$; $-A_b^{acd} + \alpha^{[dc]} \delta_b^a - \alpha^{a[c} \delta_b^{d]} - \frac{1}{2}\alpha^a B^{cd}{}_b = 0$;

$-A_b^{ac0} - \alpha_o \alpha^c \delta_b^a - \alpha^{co} \delta_b^a + \alpha_0^c \delta_b^a - C^c \alpha_o \delta_b^a + \alpha^{ao} \delta_b^c + \alpha^a \alpha_o \delta_b^c = 0$;

$A_{ad}^{[b|c|h]} = 0$; $A_a^{[bcd]} = 0$; $A_{adh}^{bc} - \alpha_{ad}{}^c \delta_h^b + \alpha_{hd}{}^c \delta_a^b = 0$;

$$A_{ad}^{[bc]} - \alpha_a{}^{[c}\delta_d^{b]} - 2\alpha_a\alpha^{[c}\delta_d^{b]} + \alpha_d{}^{[c}\delta_a^{b]} - \alpha^h\alpha_h\delta_a^{[b}\delta_d^{c]} + \alpha_d\alpha^{[c}\delta_a^{b]} = 0\,;$$

$$A_a^{[bc]0} + \alpha_0^{[c}\delta_a^{b]} + \alpha_o C^{[b}\delta_a^{c]} = 0\,;$$

$$A_{acd}^{b} - \alpha_{a[c}\delta_{d]}^{b} + \alpha_a\alpha_{[d}\delta_{c]}^{b} + \alpha_{[dc]}\delta_a^b = 0\,;$$

$$A_{ac0}^{b} + \alpha_{a0}\,\delta_c^b + \alpha_a\,\alpha_o\delta_c^b - \alpha_{c0}\,\delta_a^b - \alpha_c\,\alpha_o\,\delta_a^b + \alpha_{0c}\,\delta_a^b - \alpha_o C_c\delta_a^b = 0\,.$$

Proof. By acting operator d on Theorem 3.3; item 1), we obtain:

$$d(d\omega) = d^2\omega = 0\,;$$

So, $d(C_b\omega\wedge\omega^b + C^b\omega\wedge\omega_b) = 0\,;$

$$0 = dC_b\wedge\omega\wedge\omega^b + C_b\,d\omega\wedge\omega^b - C_b\omega\wedge d\omega^b + dC^b\wedge\omega\wedge\omega_b + C^b\,d\omega\wedge\omega_b$$
$$-C^b\omega\wedge d\omega_b\,.$$

Also, from Theorem 3.3, we get:

$$0 = dC_b\wedge\omega\wedge\omega^b + C_b\,(C_h\omega\wedge\omega^h + C^h\omega\wedge\omega_h)\wedge\omega^b + C_b\omega\wedge\theta_c^b\wedge\omega^c$$
$$-C_b\omega\wedge\alpha^b\omega^c\wedge\omega_c + C_b\omega\wedge\alpha^c\omega^b\wedge\omega_c + dC^b\wedge\omega\wedge\omega_b + C^b\omega\wedge\alpha_c\,\omega_b\wedge\omega^c$$
$$+C^b\,(C_h\omega\wedge\omega^h + C^h\omega\wedge\omega_h)\wedge\omega_b - C^b\omega\wedge\theta_b^c\wedge\omega_c - C^b\omega\wedge\alpha_b\,\omega_c\wedge\omega^c\,;$$
$$0 = dC_b\wedge\omega\wedge\omega^b + C_{[b}C_{h]}\omega\wedge\omega^h\wedge\omega^b + C_bC^h\omega\wedge\omega_h\wedge\omega^b + C_b\omega\wedge\theta_c^b\wedge\omega^c$$
$$-C_b\omega\wedge\alpha^b\omega^c\wedge\omega_c + C_b\omega\wedge\alpha^c\omega^b\wedge\omega_c + dC^b\wedge\omega\wedge\omega_b - C_b\,C^h\omega\wedge\omega_h\wedge\omega^b$$
$$+C^{[b}C^{h]}\omega\wedge\omega_h\wedge\omega_b - C^b\omega\wedge\theta_b^c\wedge\omega_c - C^b\omega\wedge\alpha_b\,\omega_c\wedge\omega^c + C^b\omega\wedge\alpha_c\,\omega_b\wedge\omega^c\,;$$

Since $C_{[b}C_{h]} = \frac{1}{2}(C_bC_h - C_hC_b) = 0$. So, $C^{[b}C^{h]} = 0$, and we have :

$$0 = dC_b\wedge\omega\wedge\omega^b - C_d\theta_b^d\wedge\omega\wedge\omega^b - C_b\omega\wedge\alpha^b\omega^c\wedge\omega_c + C_b\omega\wedge\alpha^c\omega^b\wedge\omega_c$$
$$+dC^b\wedge\omega\wedge\omega_b + C^d\,\theta_d^b\wedge\omega\wedge\omega_b - C^b\omega\wedge\alpha_b\,\omega_c\wedge\omega^c + C^b\omega\wedge\alpha_c\,\omega_b\wedge\omega^c\,;$$
$$0 = \left(dC_b - C_d\theta_b^d\right)\wedge\omega\wedge\omega^b - C_b\omega\wedge\alpha^b\omega^c\wedge\omega_c + C_b\omega\wedge\alpha^c\omega^b\wedge\omega_c$$
$$+\left(dC^b + C^d\,\theta_d^b\right)\wedge\omega\wedge\omega_b - C^b\omega\wedge\alpha_b\,\omega_c\wedge\omega^c + C^b\omega\wedge\alpha_c\,\omega_b\wedge\omega^c\,.$$

But $dC_b - C_d\,\theta_b^d = C_{bh}^d\theta_d^h + C_{bd}\,\omega^d + C_b{}^d\,\omega_d + C_{bo}\,\omega$, and

$dC^b + C^d\,\theta_d^b = C_d^{bh}\theta_h^d + C^{bd}\,\omega_d + C^b{}_d\,\omega^d + C^{bo}\omega$. So, we have

$$0 = \left(C_{bh}^d\theta_d^h + C_{bd}\,\omega^d + C_b{}^d\,\omega_d + C_{bo}\,\omega\right)\wedge\omega\wedge\omega^b - C_b\omega\wedge\alpha^b\omega^c\wedge\omega_c$$

$$+C_b\omega\ \wedge\alpha^c\omega^b\wedge\omega_c\ +\left(C_d^{bh}\theta_h^d+C^{bd}\ \omega_d+C^b_{\ d}\ \omega^d+C^{bo}\omega\right)\wedge\omega\wedge\ \omega_b$$
$$-C^b\omega\wedge\alpha_b\ \omega_c\wedge\omega^c+C^b\omega\wedge\alpha_c\ \omega_b\wedge\omega^c\ ;$$
$$0=C_{bh}^d\theta_d^h\wedge\omega\wedge\ \omega^b+C_{[bd]}\ \omega^d\wedge\omega\wedge\ \omega^b+C_b^{\ d}\ \omega_d\wedge\omega\wedge\ \omega^b-C_b\omega\ \wedge\alpha^b\omega^c\wedge\omega_c$$
$$+C_b\omega\ \wedge\alpha^c\omega^b\wedge\omega_c\ +C_d^{bh}\theta_h^d\wedge\omega\wedge\ \omega_b+C^{[bd]}\ \omega_d\wedge\omega\wedge\ \omega_b+C^b_{\ d}\ \omega^d\wedge\omega\wedge\ \omega_b$$
$$-C^b\omega\wedge\alpha_b\ \omega_c\wedge\omega^c+C^b\omega\wedge\alpha_c\ \omega_b\wedge\omega^c\ ;$$
$$0=C_{bh}^d\theta_d^h\wedge\omega\wedge\ \omega^b+C_{[bd]}\ \omega^d\wedge\omega\wedge\ \omega^b+C_d^{bh}\theta_h^d\wedge\omega\wedge\ \omega_b+C^{[bd]}\ \omega_d\wedge\omega\wedge\ \omega_b$$
$$+C_b^{\ d}\ \omega_d\wedge\omega\wedge\ \omega^b-C_h\alpha^h\delta_b^d\omega_d\wedge\omega\wedge\ \omega_c+C_b\alpha^d\omega_d\wedge\omega\wedge\omega^b-C^d_{\ b}\ \omega^d\wedge\omega\wedge\ \omega_b$$
$$+C^h\alpha_h\delta_b^d\ \omega_d\wedge\omega\wedge\omega^b-C^d\alpha_b\omega_d\wedge\omega\wedge\omega^b\ .$$

Thus, the above implies that:

$$C_{bh}^d=0,\quad C_d^{bh}=0,\quad C_{[bd]}=0,\quad C^{[bd]}=0\ ;$$

$$C_b^{\ d}-C_h\alpha^h\delta_b^d+C_b\alpha^d-C^d_{\ b}+C^h\alpha_h\delta_b^d-C^d\alpha_b=0\ .$$

Now, from Theorem 3.3; item 2), we get:

$$d(\omega^a)=d^2\omega^a=0\ ;$$

$$d(-\theta_b^a\wedge\ \omega^b+\alpha^a\omega^b\wedge\omega_b-\alpha^b\omega^a\wedge\omega_b+\ \alpha_o\ \omega\wedge\omega^a)=0\ ;$$

$$0=-d\theta_b^a\wedge\omega^b+\theta_b^a\wedge d\omega^b+d\alpha^a\wedge\omega^b\wedge\omega_b+\alpha^a\ d\omega^b\wedge\omega_b-\alpha^a\omega^b\wedge d\omega_b$$
$$-\ d\alpha^b\wedge\omega^a\wedge\omega_b-\alpha^b\ d\omega^a\wedge\omega_b+\ \alpha^b\omega^a\wedge d\omega_b+\ \ d\alpha_o\wedge\omega\wedge\omega^a+\alpha_o\ d\omega\wedge\omega^a$$
$$-\alpha_o\ \omega\wedge d\omega^a\ .$$

So, according to Theorem 3.3, we get:

$$0=-d\theta_b^a\wedge\omega^b+\theta_b^a\wedge(-\theta_c^b\wedge\ \omega^c+\alpha^b\omega^c\wedge\omega_c-\alpha^c\omega^b\wedge\omega_c+\ \alpha_o\ \omega\wedge\omega^b)$$
$$+\ d\alpha^a\wedge\omega^b\wedge\omega_b+\alpha^a\ (-\theta_c^b\wedge\ \omega^c+\alpha^b\omega^c\wedge\omega_c-\alpha^c\omega^b\wedge\omega_c+\ \alpha_o\ \omega\wedge\omega^b)\wedge\omega_b$$
$$-\ \alpha^a\omega^b\wedge(\theta_b^c\wedge\ \omega_c+\alpha_b\ \omega_c\wedge\omega^c-\alpha_c\ \omega_b\wedge\omega^c+\ \alpha_o\ \omega\wedge\omega_b)-\ d\alpha^b\wedge\omega^a\wedge\omega_b$$
$$-\ \alpha^b\ (-\theta_c^a\wedge\ \omega^c+\alpha^a\omega^c\wedge\omega_c-\alpha^c\omega^a\wedge\omega_c+\ \alpha_o\ \omega\wedge\omega^a)\wedge\omega_b+d\alpha_o\wedge\omega\wedge\omega^a$$
$$+\ \alpha^b\omega^a\wedge(\theta_b^c\wedge\ \omega_c+\alpha_b\ \omega_c\wedge\omega^c-\alpha_c\ \omega_b\wedge\omega^c+\ \alpha_o\ \omega\wedge\omega_b)$$
$$+\ \alpha_o\ (C_b\omega\wedge\omega^b+C^b\omega\wedge\omega_b)\wedge\omega^a-\alpha_o\ \omega\wedge(-\theta_b^a\wedge\ \omega^b+\alpha^a\omega^b\wedge\omega_b$$
$$-\ \alpha^b\omega^a\wedge\omega_b+\ \alpha_o\ \omega\wedge\omega^a\ )\ ;$$
$$0=-d\theta_b^a\wedge\omega^b-\theta_b^a\wedge\theta_c^b\wedge\ \omega^c+\theta_b^a\wedge\alpha^b\omega^c\wedge\omega_c-\ \theta_b^a\wedge\alpha^c\omega^b\wedge\omega_c+\theta_b^a\wedge\ \alpha_o\ \omega\wedge\omega^b$$
$$+d\alpha^a\wedge\omega^b\wedge\omega_b-\alpha^a\theta_c^b\wedge\ \omega^c\wedge\omega_b+\alpha^a\alpha^b\omega^c\wedge\omega_c\wedge\omega_b-\alpha^a\alpha^c\omega^b\wedge\omega_c\wedge\omega_b$$

$+\alpha^a\alpha_o\,\omega\wedge\omega^b\wedge\omega_b-\alpha^a\omega^b\wedge\theta_b^c\wedge\,\omega_c-\alpha^a\omega^b\wedge\alpha_b\,\omega_c\wedge\omega^c+\alpha^a\omega^b\wedge\alpha_c\,\omega_b\wedge\omega^c$

$-\alpha^a\omega^b\wedge\alpha_o\,\omega\wedge\omega_b-\,d\alpha^b\wedge\omega^a\wedge\omega_b+\alpha^b\theta_c^a\wedge\,\omega^c\wedge\omega_b-\alpha^b\alpha^a\omega^c\wedge\omega_c\wedge\omega_b$

$+\alpha^{[b}\alpha^{c]}\omega^a\wedge\omega_c\wedge\omega_b-\alpha^b\alpha_o\,\omega\wedge\omega^a\wedge\omega_b+\alpha^b\omega^a\wedge\theta_b^c\wedge\,\omega_c+\alpha^b\omega^a\wedge\alpha_b\,\omega_c\wedge\omega^c$

$-\alpha^b\omega^a\wedge\alpha_c\,\omega_b\wedge\omega^c+\,\alpha^b\omega^a\wedge\alpha_o\,\omega\wedge\omega_b+\,\,d\alpha_o\wedge\omega\wedge\omega^a+\,\alpha_oC_b\omega\wedge\omega^b\wedge\omega^a$

$+\alpha_oC^b\omega\wedge\omega_b\wedge\omega^a+\alpha_o\,\omega\wedge\theta_b^a\wedge\,\omega^b-\alpha_o\,\omega\wedge\alpha^a\omega^b\wedge\omega_b+\alpha_o\,\omega\wedge\alpha^b\omega^a\wedge\omega_b$

$-\alpha_o\,\omega\wedge\,\alpha_o\,\omega\wedge\omega^a\,;$

$0=-(d\theta_b^a+\theta_c^a\wedge\theta_b^c)\wedge\,\omega^b+(d\alpha^a+\alpha^c\theta_c^a)\wedge\omega^b\wedge\omega_b+\alpha^a\alpha^b\omega^c\wedge\omega_c\wedge\omega_b$

$+2\alpha^a\alpha_c\,\omega^b\wedge\,\omega_b\wedge\omega^c+\alpha^a\alpha_o\,\omega\wedge\omega^b\wedge\omega_b-(d\alpha^b+\alpha^c\theta_c^b)\wedge\omega^a\wedge\omega_b$

$-\alpha^b\alpha_o\,\omega\wedge\omega^a\wedge\omega_b+\alpha^b\alpha_b\,\omega^a\wedge\,\omega_c\wedge\omega^c-\alpha^b\alpha_c\,\omega^a\wedge\,\omega_b\wedge\omega^c$

$+d\alpha_o\wedge\omega\wedge\omega^a+\,\alpha_oC_b\omega\wedge\omega^b\wedge\omega^a+\alpha_oC^b\omega\wedge\omega_b\wedge\omega^a\,.$

Set $d\theta_b^a+\theta_b^a\wedge\theta_c^b=A_{bcf}^{adh}\,\theta_d^c\wedge\theta_h^f+A_{bch}^{ad}\,\theta_d^c\wedge\omega^h+A_{bc}^{adh}\,\theta_d^c\wedge\omega_h+A_{bc0}^{ad}\,\theta_d^c\wedge\omega$

$+A_{bcd}^a\,\omega^c\wedge\omega^d+A_{bc}^{ad}\,\omega^c\wedge\omega_d+A_{bc0}^a\,\omega^c\wedge\omega+A_b^{acd}\,\omega_c\wedge\omega_d+A_b^{ac0}\omega_c\wedge\omega$, and

$d\alpha_o=\alpha_{0c}^d\,\theta_d^c+\alpha_{0c}\,\omega^c+\alpha_0^c\omega_c+\alpha_{00}\omega\,.$

Now , let $d\alpha^b+\alpha^c\,\theta_c^b=\alpha_d^{bc}\,\theta_c^d+\alpha_c^b\,\omega^c+\alpha^{bc}\,\omega_c+\alpha^{bo}\omega\,.$

Notice that all coefficients added above are suitable smooth map. Then

$0=-(A_{bcf}^{adh}\,\theta_d^c\wedge\theta_h^f+A_{bch}^{ad}\,\theta_d^c\wedge\omega^h+A_{bc}^{adh}\,\theta_d^c\wedge\omega_h+A_{bc0}^{ad}\,\theta_d^c\wedge\omega+A_{bcd}^a\,\omega^c\wedge\omega^d$

$+A_{bc}^{ad}\,\omega^c\wedge\omega_d+A_{bc0}^a\,\omega^c\wedge\omega+A_b^{acd}\,\omega_c\wedge\omega_d+A_b^{ac0}\omega_c\wedge\omega)\wedge\,\omega^b+\alpha^a\alpha^b\omega^c\wedge\omega_c\wedge\omega_b$

$+(\alpha_d^{ac}\,\theta_c^d+\alpha_c^a\,\omega^c+\alpha^{ac}\,\omega_c+\alpha^{ao}\omega)\wedge\omega^b\wedge\omega_b+2\alpha^a\alpha_c\,\omega^b\wedge\,\omega_b\wedge\omega^c$

$+\alpha^a\alpha_o\,\omega\wedge\omega^b\wedge\omega_b-\left(\alpha_d^{bc}\,\theta_c^d+\alpha_c^b\,\omega^c+\alpha^{bc}\,\omega_c+\alpha^{bo}\omega\right)\wedge\omega^a\wedge\omega_b$

$-\alpha^b\alpha_o\,\omega\wedge\omega^a\wedge\omega_b+\alpha^b\alpha_b\,\omega^a\wedge\,\omega_c\wedge\omega^c-\alpha^b\alpha_c\,\omega^a\wedge\,\omega_b\wedge\omega^c$

$+\left(\alpha_{0c}^d\,\theta_d^c+\alpha_{0c}\,\omega^c+\alpha_0^c\omega_c+\alpha_{00}\omega\right)\wedge\omega\wedge\omega^a+\,\alpha_oC_b\omega\wedge\omega^b\wedge\omega^a+\alpha_oC^b\omega\wedge\omega_b\wedge\omega^a\,;$

$0=-A_{bcf}^{adh}\,\theta_d^c\wedge\theta_h^f\wedge\,\omega^b-A_{[b|c|h]}^{ad}\,\theta_d^c\wedge\omega^h\wedge\,\omega^b-A_{bc}^{adh}\,\theta_d^c\wedge\omega_h\wedge\,\omega^b-A_{bc0}^{ad}\,\theta_d^c\wedge\omega\wedge\,\omega^b$

$-A_{[bcd]}^a\,\omega^c\wedge\omega^d\wedge\,\omega^b-A_{[bc]}^{ad}\,\omega^c\wedge\omega_d\wedge\,\omega^b-A_{[bc]0}^a\,\omega^c\wedge\omega\wedge\,\omega^b-A_b^{acd}\,\omega_c\wedge\omega_d\wedge\,\omega^b$

$-A_b^{ac0}\omega_c\wedge\omega\wedge\,\omega^b+\alpha_d^{ac}\,\theta_c^d\wedge\omega^b\wedge\omega_b+\alpha_c^a\,\omega^c\wedge\omega^b\wedge\omega_b+\alpha^{ac}\,\omega_c\wedge\omega^b\wedge\omega_b$

$+\alpha^{ao}\omega\wedge\omega^b\wedge\omega_b+\alpha^a\alpha^b\omega^c\wedge\omega_c\wedge\omega_b+2\alpha^a\alpha_c\,\omega^b\wedge\,\omega_b\wedge\omega^c+\alpha^a\alpha_o\,\omega\wedge\omega^b\wedge\omega_b$

$-\alpha_d^{bc}\,\theta_c^d\wedge\omega^a\wedge\omega_b-\alpha_c^b\,\omega^c\wedge\omega^a\wedge\omega_b-\alpha^{[bc]}\,\omega_c\wedge\omega^a\wedge\omega_b-\alpha^{bo}\omega\wedge\omega^a\wedge\omega_b$

$-\alpha^b\alpha_o\,\omega\wedge\omega^a\wedge\omega_b+\alpha^b\alpha_b\,\omega^a\wedge\,\omega_c\wedge\omega^c-\alpha^b\alpha_c\,\omega^a\wedge\,\omega_b\wedge\omega^c+\alpha_{0c}^d\,\theta_d^c\wedge\omega\wedge\omega^a$

$$+\alpha_{0c}\,\omega^c\wedge\omega\wedge\omega^a+\alpha_0^c\,\omega_c\wedge\omega\wedge\omega^a+\alpha_{00}\,\omega\wedge\omega\wedge\omega^a+\ \alpha_o C_b\omega\wedge\omega^b\wedge\omega^a$$
$$+\alpha_o C^b\omega\wedge\omega_b\wedge\omega^a\,;$$
$$0=-A^{adh}_{bcf}\,\theta^c_d\wedge\theta^f_h\wedge\ \omega^b-A^{ad}_{[b|c|h]}\,\theta^c_d\wedge\omega^h\wedge\ \omega^b-A^a_{[bcd]}\,\omega^c\wedge\omega^d\wedge\ \omega^b$$
$$+\left(-A^{adh}_{bc}\,\theta^c_d\wedge\omega_h\wedge\ \omega^b+\alpha^{hd}_c\theta^c_d\wedge\delta^a_b\,\omega^b\wedge\omega_h-\alpha^{ad}_c\,\theta^c_d\wedge\delta^h_b\omega^b\wedge\omega_h\right)$$
$$+\left(-A^{ad}_{bc0}\,\theta^c_d\wedge\omega\wedge\ \omega^b+\alpha^d_{0c}\,\theta^c_d\wedge\omega\wedge\delta^a_b\omega^b\right)+(-A^{ad}_{[bc]}\,\omega^c\wedge\omega_d\wedge\ \omega^b$$
$$+\alpha^d\alpha_{[c}\delta^a_{b]}\,\omega^b\wedge\ \omega_d\wedge\omega^c+\alpha^d_{[c}\delta^a_{b]}\,\omega^c\wedge\omega^b\wedge\omega_b-\alpha^a_{[c}\delta^d_{b]}\,\omega^c\wedge\omega^b\wedge\omega_d$$
$$-2\alpha^a\alpha_{[c}\,\delta^d_{b]}\omega^b\wedge\ \omega_b\wedge\omega^c-\alpha^h\alpha_h\delta^a_{[b}\delta^d_{c]}\,\omega^b\wedge\ \omega_d\wedge\omega^c\,)+(-A^{acd}_b\,\omega_c\wedge\omega_d\wedge\ \omega^b$$
$$+\alpha^{[dc]}\delta^a_b\,\omega_c\wedge\omega^b\wedge\omega_d-\alpha^{a[c}\delta^{d]}_b\,\omega_c\wedge\omega^b\wedge\omega_d+\alpha^a\alpha^{[d}\delta^{c]}_b\omega^b\wedge\omega_c\wedge\omega_d\,)$$
$$+(-A^{ac0}_b\omega_c\wedge\omega\wedge\ \omega^b-\alpha_o\alpha^c\delta^a_b\,\omega\wedge\omega^b\wedge\omega_c-\alpha^{co}\delta^a_b\omega\wedge\omega^b\wedge\omega_c\ +\alpha^c_0\delta^a_b\,\omega_c\wedge\omega\wedge\omega^b$$
$$-C^c\alpha_o\delta^a_b\omega\wedge\omega_c\wedge\omega^b+\alpha^{ao}\delta^c_b\omega\wedge\omega^b\wedge\omega_c+\alpha^a\alpha_o\delta^c_b\,\omega\wedge\omega^b\wedge\omega_c\,)$$
$$+\left(-A^a_{[bc]0}\,\omega^c\wedge\omega\wedge\ \omega^b+\alpha_{0[c}\delta^a_{b]}\,\omega^c\wedge\omega\wedge\omega^b+\ \alpha_o C_{[b}\delta^a_{c]}\omega\wedge\omega^b\wedge\omega^c\right);$$

$$\begin{bmatrix} A^{adh}_{bcf}=0\quad,\quad A^{ad}_{[b|c|h]}=0\ ,\quad A^a_{[bcd]}=0\,; \\ -A^{adh}_{bc}+\alpha^{hd}_c\delta^a_b-\alpha^{ad}_c\,\delta^h_b=0\qquad,\qquad -A^{ad}_{bc0}+\alpha^d_{0c}\delta^a_b\ =0\,; \\ -A^{ad}_{[bc]}-\frac{1}{2}\alpha^d B_{bc}{}^a\ +\alpha^d_{[c}\delta^a_{b]}\ -\alpha^a_{[c}\delta^d_{b]}+\alpha^a B_{bc}{}^d-\alpha^h\alpha_h\delta^a_{[b}\delta^d_{c]}\ =0\,; \\ -A^a_{[bc]0}\ +\alpha_{0[c}\delta^a_{b]}\ +\ \alpha_o C_{[b}\delta^a_{c]}=0\,; \\ -A^{acd}_b+\alpha^{[dc]}\delta^a_b-\alpha^{a[c}\delta^{d]}_b-\frac{1}{2}\alpha^a B^{cd}{}_b=0\,; \\ -A^{ac0}_b-\alpha_o\alpha^c\delta^a_b\ -\alpha^{co}\delta^a_b+\alpha^c_0\delta^a_b\ -C^c\alpha_o\delta^a_b+\alpha^{ao}\delta^c_b+\alpha^a\alpha_o\delta^c_b\ =0\,. \end{bmatrix}$$
(3.5)

Now, from Theorem 3.3; item 3), we obtain:

$$d(d\omega_a)=d^2\omega_a=0\,;$$

$$d(\theta^b_a\wedge\ \omega_b+\alpha_a\omega_b\wedge\omega^b-\alpha_b\omega_a\wedge\omega^b+\ \alpha_o\,\omega\wedge\omega_a)=0\,;$$

$$0=d\theta^b_a\wedge\ \omega_b-\theta^b_a\wedge\ d\omega_b+d\alpha_a\wedge\omega_b\wedge\omega^b+\alpha_a d\omega_b\wedge\omega^b-\alpha_a\omega_b\wedge d\omega^b$$
$$-d\alpha_b\wedge\omega_a\wedge\omega^b-\alpha_b d\omega_a\wedge\omega^b+\alpha_b\omega_a\wedge\ d\omega^b+d\alpha_o\wedge\omega\wedge\omega_a+\alpha_o d\omega\wedge\omega_a$$
$$-\alpha_o\,\omega\wedge d\omega_a\,.$$

So, again from Theorem 3.3, we have:

$$0 = d\theta_a^b \wedge \omega_b - \theta_a^b \wedge (\theta_b^c \wedge \omega_c + \alpha_b\, \omega_c \wedge \omega^c - \alpha_c\, \omega_b \wedge \omega^c + \alpha_o\, \omega \wedge \omega_b)$$
$$+ d\alpha_a \wedge \omega_b \wedge \omega^b + \alpha_a(\theta_b^c \wedge \omega_c + \alpha_b\, \omega_c \wedge \omega^c - \alpha_c\, \omega_b \wedge \omega^c + \alpha_o\, \omega \wedge \omega_b) \wedge \omega^b$$
$$-\alpha_a\omega_b \wedge (-\theta_c^b \wedge \omega^c + \alpha^b\omega^c \wedge \omega_c - \alpha^c\omega^b \wedge \omega_c + \alpha_o\, \omega \wedge \omega^b) - d\alpha_b \wedge \omega_a \wedge \omega^b$$
$$-\alpha_b(\theta_a^c \wedge \omega_c + \alpha_a\omega_c \wedge \omega^c - \alpha_c\omega_a \wedge \omega^c + \alpha_o\, \omega \wedge \omega_a) \wedge \omega^b + d\alpha_o \wedge \omega \wedge \omega_a$$
$$+\alpha_b\omega_a \wedge (-\theta_c^b \wedge \omega^c + \alpha^b\omega^c \wedge \omega_c - \alpha^c\omega^b \wedge \omega_c + \alpha_o\, \omega \wedge \omega^b) + \alpha_o(C_b\omega \wedge \omega^b$$
$$+C^b\omega \wedge \omega_b) \wedge \omega_a - \alpha_o\, \omega \wedge (\theta_a^b \wedge \omega_b + \alpha_a\omega_b \wedge \omega^b - \alpha_b\omega_a \wedge \omega^b + \alpha_o\, \omega \wedge \omega_a) ;$$

$$0 = d\theta_a^b \wedge \omega_b - \theta_a^b \wedge \theta_b^c \wedge \omega_c - \theta_a^b \wedge \alpha_b\, \omega_c \wedge \omega^c + \theta_a^b \wedge \alpha_c\, \omega_b \wedge \omega^c - \theta_a^b \wedge \alpha_o\, \omega \wedge \omega_b$$
$$+d\alpha_a \wedge \omega_b \wedge \omega^b + \alpha_a\theta_b^c \wedge \omega_c \wedge \omega^b + \alpha_a\alpha_b\, \omega_c \wedge \omega^c \wedge \omega^b - \alpha_a\alpha_c\, \omega_b \wedge \omega^c \wedge \omega^b$$
$$+\alpha_a\, \alpha_o\, \omega \wedge \omega_b \wedge \omega^b + \alpha_a\omega_b \wedge \theta_c^b \wedge \omega^c - \alpha_a\omega_b \wedge \alpha^b\omega^c \wedge \omega_c + \alpha_a\omega_b \wedge \alpha^c\omega^b \wedge \omega_c$$
$$-\alpha_a\omega_b \wedge \alpha_o\, \omega \wedge \omega^b - d\alpha_b \wedge \omega_a \wedge \omega^b - \alpha_b\theta_a^c \wedge \omega_c \wedge \omega^b - \alpha_b\alpha_a\omega_c \wedge \omega^c \wedge \omega^b$$
$$+\alpha_{[b}\alpha_{c]}\omega_a \wedge \omega^c \wedge \omega^b - \alpha_b\, \alpha_o\, \omega \wedge \omega_a \wedge \omega^b - \alpha_b\omega_a \wedge \theta_c^b \wedge \omega^c + \alpha_b\omega_a \wedge \alpha^b\omega^c \wedge \omega_c$$
$$-\alpha_b\omega_a \wedge \alpha^c\omega^b \wedge \omega_c + \alpha_b\omega_a \wedge \alpha_o\, \omega \wedge \omega^b + d\alpha_o \wedge \omega \wedge \omega_a + \alpha_o C_b\omega \wedge \omega^b \wedge \omega_a$$
$$+\alpha_o C^b\omega \wedge \omega_b \wedge \omega_a - \alpha_o\, \omega \wedge \theta_a^b \wedge \omega_b - \alpha_o\, \omega \wedge \alpha_a\omega_b \wedge \omega^b$$
$$+\alpha_o\, \omega \wedge \alpha_b\omega_a \wedge \omega^b - \alpha_o\, \omega \wedge \alpha_o\, \omega \wedge \omega_a ;$$

$$0 = (d\theta_a^b - \theta_a^c \wedge \theta_c^b) \wedge \omega_b + (d\alpha_a - \alpha_c\theta_a^c) \wedge \omega_b \wedge \omega^b + \alpha_a\alpha_b\, \omega_c \wedge \omega^c \wedge \omega^b$$
$$+\alpha_a\, \alpha_o\, \omega \wedge \omega_b \wedge \omega^b - 2\alpha_a\omega_b \wedge \alpha^b\omega^c \wedge \omega_c - (d\alpha_b - \alpha_c\, \theta_b^c) \wedge \omega_a \wedge \omega^b$$
$$-\alpha_b\, \alpha_o\, \omega \wedge \omega_a \wedge \omega^b + \alpha_b\omega_a \wedge \alpha^b\omega^c \wedge \omega_c - \alpha_b\omega_a \wedge \alpha^c\omega^b \wedge \omega_c + d\alpha_o \wedge \omega \wedge \omega_a$$
$$+\alpha_o C_b\omega \wedge \omega^b \wedge \omega_a + \alpha_o C^b\omega \wedge \omega_b \wedge \omega_a .$$

Since $d\theta_a^b - \theta_a^c \wedge \theta_c^b = A_{ad}^{bch}\, \theta_c^d \wedge \omega_h + A_{adh}^{bc}\, \theta_c^d \wedge \omega^h + A_{ad0}^{bc}\, \theta_c^d \wedge \omega + A_a^{bcd}\, \omega_c \wedge \omega_d$

$$+A_{ad}^{bc}\, \omega_c \wedge \omega^d + A_{a0}^{bc}\, \omega_c \wedge \omega + A_{acd}^{b}\, \omega^c \wedge \omega^d + A_{ac0}^{b}\, \omega^c \wedge \omega ;$$

$$d\alpha_o = \alpha_{0c}^d\, \theta_d^c + \alpha_{0c}\, \omega^c + \alpha_0^c\omega_c + \alpha_{00}\omega ;$$

$$d\alpha_b - \alpha_c\, \theta_b^c = \alpha_{bc}{}^d\, \theta_d^c + \alpha_b{}^c\, \omega_c + \alpha_{bc}\, \omega^c + \alpha_{b0}\, \omega .$$

Then we get:

$$0 = (A_{ad}^{bch}\, \theta_c^d \wedge \omega_h + A_{adh}^{bc}\, \theta_c^d \wedge \omega^h + A_{ad0}^{bc}\, \theta_c^d \wedge \omega + A_a^{bcd}\, \omega_c \wedge \omega_d + A_{ad}^{bc}\, \omega_c \wedge \omega^d$$

$$+A_{a0}^{bc}\,\omega_c\wedge\omega+A_{acd}^{b}\,\omega^c\wedge\omega^d+A_{ac0}^{b}\,\omega^c\wedge\omega)\wedge\ \omega_b+\alpha_a\alpha_b\,\omega_c\wedge\omega^c\wedge\omega^b$$

$$+(\alpha_{ac}{}^{d}\,\theta_d^c+\alpha_a{}^{c}\,\omega_c+\alpha_{ac}\,\omega^c+\alpha_{a0}\,\omega)\wedge\omega_b\wedge\omega^b+\alpha_a\,\alpha_o\,\omega\wedge\omega_b\wedge\omega^b$$

$$-2\alpha_a\omega_b\wedge\alpha^b\omega^c\wedge\omega_c-\left(\alpha_{bc}{}^{d}\,\theta_d^c+\alpha_b{}^{c}\,\omega_c+\alpha_{bc}\,\omega^c+\alpha_{b0}\omega\right)\wedge\omega_a\wedge\omega^b$$

$$-\alpha_b\,\alpha_o\,\omega\wedge\omega_a\wedge\omega^b+\alpha_b\omega_a\wedge\alpha^b\omega^c\wedge\omega_c-\alpha_b\omega_a\wedge\alpha^c\omega^b\wedge\omega_c+\alpha_oC^b\omega\wedge\omega_b\wedge\omega_a$$

$$+\left(\alpha_{0c}^{d}\,\theta_d^c+\alpha_{0c}\,\omega^c+\alpha_0^c\omega_c+\alpha_{00}\,\omega\right)\wedge\omega\wedge\omega_a+\alpha_oC_b\omega\wedge\omega^b\wedge\omega_a\,;$$

$$0=A_{ad}^{[b|c|h]}\,\theta_c^d\wedge\omega_h\wedge\ \omega_b+A_{adh}^{bc}\,\theta_c^d\wedge\omega^h\wedge\ \omega_b+A_{ad0}^{bc}\,\theta_c^d\wedge\omega\wedge\ \omega_b+\alpha_oC^b\omega\wedge\omega_b\wedge\omega_a$$

$$+A_a^{[bcd]}\,\omega_c\wedge\omega_d\wedge\ \omega_b+A_{ad}^{[bc]}\,\omega_c\wedge\omega^d\wedge\ \omega_b+A_{a0}^{[bc]}\omega_c\wedge\omega\wedge\ \omega_b+A_{acd}^{b}\,\omega^c\wedge\omega^d\wedge\ \omega_b$$

$$+A_{ac0}^{b}\,\omega^c\wedge\omega\wedge\ \omega_b\ +\alpha_{ac}{}^{d}\,\theta_d^c\wedge\omega_b\wedge\omega^b+\alpha_a{}^{c}\,\omega_c\wedge\omega_b\wedge\omega^b+\alpha_{ac}\,\omega^c\wedge\omega_b\wedge\omega^b$$

$$+\alpha_{a0}\,\omega\wedge\omega_b\wedge\omega^b+\alpha_a\alpha_b\,\omega_c\wedge\omega^c\wedge\omega^b+\alpha_a\,\alpha_o\,\omega\wedge\omega_b\wedge\omega^b-2\alpha_a\omega_b\wedge\alpha^b\omega^c\wedge\omega_c$$

$$-\alpha_{bc}{}^{d}\,\theta_d^c\wedge\omega_a\wedge\omega^b-\alpha_b{}^{c}\,\omega_c\wedge\omega_a\wedge\omega^b-\alpha_{[bc]}\,\omega^c\wedge\omega_a\wedge\omega^b-\alpha_{b0}\omega\wedge\omega_a\wedge\omega^b$$

$$-\alpha_b\,\alpha_o\,\omega\wedge\omega_a\wedge\omega^b+\alpha_b\omega_a\wedge\alpha^b\omega^c\wedge\omega_c-\alpha_b\omega_a\wedge\alpha^c\omega^b\wedge\omega_c+\alpha_{0c}^{d}\,\theta_d^c\wedge\omega\wedge\omega_a$$

$$+\alpha_{0c}\,\omega^c\wedge\omega\wedge\omega_a+\alpha_0^c\,\omega_c\wedge\omega\wedge\omega_a+\alpha_{00}\,\omega\wedge\omega\wedge\omega_a+\alpha_oC_b\omega\wedge\omega^b\wedge\omega_a\,;$$

$$0=\left(A_{adh}^{bc}\,\theta_c^d\wedge\omega^h\wedge\ \omega_b-\alpha_{ad}{}^{c}\delta_h^b\,\theta_c^d\wedge\omega^h\wedge\omega_b+\alpha_{hd}{}^{c}\delta_a^b\,\theta_c^d\wedge\omega^h\wedge\omega_b\right)$$

$$+A_{ad}^{[b|c|h]}\,\theta_c^d\wedge\omega_h\wedge\ \omega_b+\left(A_{ad0}^{bc}\,\theta_c^d\wedge\omega\wedge\ \omega_b+\alpha_{0d}^{c}\delta_a^b\,\theta_c^d\wedge\omega\wedge\omega_b\right)$$

$$+(\,A_{ad}^{[bc]}\,\omega_c\wedge\omega^d\wedge\ \omega_b-\alpha_a{}^{[c}\delta_d^{b]}\,\omega_c\wedge\omega^d\wedge\omega_b-2\alpha_a\alpha^{[c}\delta_d^{b]}\omega_c\wedge\omega^d\wedge\omega_b$$

$$+\alpha_d{}^{[c}\delta_a^{b]}\,\omega_c\wedge\omega^d\wedge\omega_b-\alpha^h\alpha_h\delta_a^{[b}\delta_d^{c]}\omega_c\wedge\omega^d\wedge\omega_b+\alpha_d\alpha^{[c}\delta_a^{b]}\omega_c\wedge\omega^d\wedge\omega_b\,)$$

$$+\left(A_{a0}^{[bc]}\omega_c\wedge\omega\wedge\ \omega_b+\alpha_0^{[c}\delta_a^{b]}\,\omega_c\wedge\omega\wedge\omega_b+\alpha_oC^{[b}\delta_a^{c]}\omega_c\wedge\omega\wedge\omega_b\right)$$

$$+(\,A_{acd}^{b}\,\omega^c\wedge\omega^d\wedge\ \omega_b-\alpha_{a[c}\delta_{d]}^{b}\,\omega^c\wedge\omega^d\wedge\omega_b+\alpha_a\alpha_{[d}\delta_{c]}^{b}\omega^c\wedge\omega^d\wedge\ \omega_b$$

$$+\alpha_{[dc]}\delta_a^b\,\omega^c\wedge\omega^d\wedge\omega_b\,)+(\,A_{ac0}^{b}\,\omega^c\wedge\omega\wedge\ \omega_b+\alpha_{a0}\,\delta_c^b\,\omega^c\wedge\ \omega\wedge\omega_b$$

$$+\alpha_a\,\alpha_o\delta_c^b\,\omega^c\wedge\,\omega\wedge\omega_b-\alpha_{c0}\,\delta_a^b\,\omega^c\wedge\,\omega\wedge\omega_b-\alpha_c\,\alpha_o\,\delta_a^b\,\omega^c\wedge\,\omega\wedge\omega_b$$

$$+\alpha_{0c}\,\delta_a^b\,\omega^c\wedge\,\omega\wedge\omega_b-\alpha_oC_c\delta_a^b\,\omega^c\wedge\,\omega\wedge\omega_b\,)+A_a^{[bcd]}\,\omega_c\wedge\omega_d\wedge\,\omega_b\,;$$

$$\left[\begin{array}{c} A_{adh}^{bcf}=0 \quad , \quad A_{ad}^{[b|c|h]}=0 \quad , \quad A_a^{[bcd]}=0\,; \\ A_{adh}^{bc}-\alpha_{ad}{}^{c}\delta_h^b+\alpha_{hd}{}^{c}\delta_a^b=0 \quad , \quad A_{ad0}^{bc}+\alpha_{0d}^{c}\delta_a^b=0\,; \\ A_{ad}^{[bc]}-\alpha_a{}^{[c}\delta_d^{b]}-2\alpha_a\alpha^{[c}\delta_d^{b]}+\alpha_d{}^{[c}\delta_a^{b]}-\alpha^h\alpha_h\delta_a^{[b}\delta_d^{c]}+\alpha_d\alpha^{[c}\delta_a^{b]}=0\,; \\ A_{a0}^{[bc]}+\alpha_0^{[c}\delta_a^{b]}+\alpha_oC^{[b}\delta_a^{c]}=0\,; \\ A_{acd}^{b}-\alpha_{a[c}\delta_{d]}^{b}+\alpha_a\alpha_{[d}\delta_{c]}^{b}+\alpha_{[dc]}\delta_a^b=0\,; \\ A_{ac0}^{b}+\alpha_{a0}\,\delta_c^b+\alpha_a\,\alpha_o\delta_c^b-\alpha_{c0}\,\delta_a^b-\alpha_c\,\alpha_o\,\delta_a^b+\alpha_{0c}\,\delta_a^b-\alpha_oC_c\delta_a^b=0\,. \end{array}\right]$$

(3.6)

From equations (3.5) and (3.6), we get:

$-A_{bc0}^{ad}+\alpha_{0c}^{d}\delta_b^a=0$ and $A_{bc0}^{ad}+\alpha_{0c}^{d}\delta_a^b=0.$ Then $A_{bc0}^{ad}=\alpha_{0c}^{d}=0$. So, the results attained. ■

5. Conclusion

This study established that the locally conformal C_{12} −manifolds have a characterizing identity different from all other identities of the famous classes that discussed up to now. Also, the Cartan's structure equations of first and second groups to the locally conformal C_{12} −manifolds have rigidity that caused most researchers to avoided the study in this class. So, this article simplified that problem and we exhort the researchers to investigate these manifolds based on the results in this study.

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هندسة التحويل الكنفورمي المحلي للمنطويات - C_{12}

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المستخلص

هذا البحث يتعامل مع فئة التحويل الكنفورمي المحلي للمنطويات - C_{12} حيث ان هذه المعالجة انتجت المتطابقة المميزة لهذه الفئة. بالإضافة فان المجموعة الأولى من معادلات كارتان التركيبية قد تم تحديدها بعد ان تم تحديد مركبات تناسر كريجنكا على البنية G – المصاحبة للتحويل الكنفورمي المحلي للمنطويات - C_{12}. اخيراً فان المجموعة الثانية من معادلات كارتان التركيبية للتحويل الكنفورمي المحلي للمنطويات - C_{12} ايضاً تم استنتاجها.

الكلمات المفتاحية: المنطوي - C_{12} ، التحويل الكنفورمي المحلي للبنية التلامسية التقريبية ، معادلات كارتان التركيبية.