The Nonlinear Fisher's Reaction-Diffusion Equation: A Novel Fractional Derivation and Numerical Simulations Associated Theorems

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ARTICLE INFO | **ABSTRACT**

Keywords

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The fractional-order derivative introduced by Caputo-Fabrizio has important tools and applications, which this research examines. Used the new derivative on the nonlinear Fisher's reaction-diffusion problem to solve the updated equation iteratively. Our methodology is robust, as shown by fixed-point theory. Various fractional-order values were simulated numerically, along with essential theorems and proofs. Fractional diffusion equations, especially when particle clouds disperse faster than classical theory predicts, substitute the second spatial derivative with a fractional derivative of order less than two. Asymmetric solutions spread faster than conventional solutions. Complex diffusion and reaction are combined in fractional reaction-diffusion equations. We provide a practical numerical method for solving these equations via operator splitting. Assess numerical solution attributes and give numerical simulations to support the method. Additionally, we investigate biological applications where the response term reflects species expansion and the diffusion term indicates movement.

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1.Introduction

One of the most widely used concepts in applied mathematics is the derivation concept, introduced to describe the rate of change of a given function and is commonly used in the modeling of real-world equations of this concept. Despite all the advantages of derivation in real-world modeling, it has been observed over time that differential equations with ordinary derivations cannot meet the needs of scientists due to the complexity that exists in real-world problems. So the concept of fractional derivations with the initial questions of the great scientist Leibniz entered the arena then a scientist named Euler took the first step in generalizing the symbolism for the derivation of arbitrary functions taking into account the fractional values for the order of the derivation, years later Lineville also pointed out the existence of the left and right derivations by proposing new ideas and then defined the fractional integral operator in the continuation of the activities of these scientists a scientist named Riemann used the Taylor series generalization to obtain a formula for the order integral used a deficit. Since fractional derivations are one of the favorite topics of scientists in all sciences, useful studies have been conducted in this field such as [3, 10, 11, 13-16]. On the other hand, the Fisher equation is one of the equations of engineering, chemistry, and physics, so there has been valuable research on it [1, 4, 5, 13-15]. In recent years, a derivation called the Caputo-Fabrizio fractional derivation has been proposed [6, 7, 8, 9-14] and scientists have used this derivation to solve differential equations according to analytical methods, in the meantime, the Sumudu method is very effective in solving these types of equations [2, 7-11-15]. The thesis also uses the Caputo- Fabrizio fractional derivation in solving the nonlinear Fisher reaction-diffusion equation according to the Sumudu method [5-18]. we presented useful and important features of the new fractional derivation. Because the main objective of this paper is to use the Caputo-Fabrizio fractional derivation to solve the nonlinear reaction-Fisher diffusion equation using analytical methods, to achieve this goal, in this part important theorems such as the existence of Sumudu transforms, the Caputo-Fabrizio fractional derivation examination theorem is proven for specific functions such as logarithmic and exponential functions. Fractional differential equations are widely used in modeling many sciences such as physics, chemistry, and engineering. In this section numerical simulation of the nonlinear equation of Fisher emission fractional differential equations are crucial for modeling various scientific phenomena. In this section, we present the numerical simulation of a nonlinear Fisher emission equation.

The equation includes a Caputo-Fabrizio fractional derivative of order gamma, represented as ${}^{CF}_{0}D^{\gamma}_{t}$ applied to the bivariate function $u(x,t)$. This is equal to alpha multiplied by the second-order partial derivative of $u(x,t)$ with respect to x, plus beta multiplied by $u(x,t)$ times one minus $u(x,t)$ raised to the power m, where m is greater than zero and gamma lies between zero and one. The initial conditions specify that $u(x,0)$ equals zero for values of x between a and b. [19,20] We explore different values of m and various fractional derivative orders. The function $u(x,t)$ depends on both x and t, with its first-order partial derivative with respect to x and its second-order partial derivative with respect to x also considered. For computational purposes, we provide a routine to handle these terms and solve the equation effectively.

2.PRELIMINARIES:

Definition 2.1:let $f \in H^1(a,b), b > a, a \in [a,b]$ then the new caputo-Fabrizio derivative of fractional order is defined as:

$$D_t^{\alpha}\big(f(t)\big) = \frac{M(\alpha)}{1-\alpha}\int_{\alpha}^{t} f'(x)\exp\left[-\alpha\frac{t-x}{1-\alpha}\right]dx \tag{1}$$

where $M(\alpha)$ is the normalization function such that $M(0) = M(1) = 1$. But, if the function does not belong to $H^1(a, b)$ then, the derivative can be reformulated as.

$$D_t^{\alpha}\big(f(t)\big) = \frac{M(\alpha)}{1-\alpha}\int_{\alpha}^{t}(f(t)-f(x))\exp\left[-\alpha\frac{t-x}{1-\alpha}\right]dx \tag{2}$$

Definition 2.2:let $u \in H^1(0,b), b > 0, 0 < \gamma < 1$, then the time fractional Caputo-Fabrizio fractional differential Operator (C-FFDO) is defined as.

$$^{CF}D_t^{\gamma}u(t) = \frac{(2-\gamma)M(\gamma)}{2(1-\gamma)}\int_0^t exp\left[-\frac{\gamma(t-s)}{1-\gamma}\right]u'(\tau)d\tau$$

$$t \geq 0, 0 < \gamma < 1\,, \tag{3}$$

With a normalization function M(γ) which is depending on $\gamma \ni M(0) = M(1) = 1$.

Definition 2.3: the Caputo-Fabrizio Fractional Derivative Operator of order 0$< \gamma <$ $1\; is\; given\; by$

$$^{CF}D_t^{\gamma}u(t) = \frac{2(1-\gamma)}{(2-\gamma)M(\gamma)}u(t) + \frac{2\gamma}{(2-\gamma)M(\gamma)}\int_0^t u(\tau)d\tau \tag{4}$$

Similar to the usual Caputo derivative, the Caputo-Fabrizio operator yields zero when u is a constant function. The main advantage of the Caputo-Fabrizio operator over the traditional Caputo operator is that the new kernel does not exhibit a singularity at $t = s$

3. THE MAIN RESULTS

Theorem 3.1. The following ordinary differential equation with Caputo- Fabrizio derivation of the fractional order [1]

$$_{t}D_t^{\propto}(t) = f(t) \qquad f(0) \neq 0$$

It has a non-obvious answer for 0 < α < 1.

Proof: To verify this result, the Laplace transform is applied to both sides of Equation (1), yielding:

$$\mathcal{L}[{}_0^{cf}D_t^{\propto}\mathrm{f(t)}](\mathrm{s}) = \mathcal{L}[\mathrm{f(t)}](\mathrm{s})$$

$$\int_0^{\infty} \exp(-st)_0^{cf}\, D_t^{\propto} f(t)dt = \mathcal{L}[f(t)](s)$$

$$\frac{M(\alpha)}{1-\alpha}\int_0^{\infty} \exp(-st)\int_0^t f'(x)\exp\left[\frac{-\alpha(t-x)}{1-\alpha}\right]dxdt = \mathcal{L}[f(t)](s)$$

On the other hand,

$$\int_0^t f'(x)\exp\left[\frac{-\alpha}{1-\alpha}(t-x)\right]dx$$

That's the same convolution of exp $(\frac{-\alpha(t)}{1-\alpha})$ $and\ f'(x)$ so, by using the convolution feature to transform Laplace we are going to have:

$$\mathcal{L}[\int_0^{\infty} f'(x)\exp\left[\frac{-\alpha}{1-\alpha}(t-x)\right]dx] = \mathcal{L}[f'(t)]\mathcal{L}\left[\exp\left[\frac{-\alpha}{1-\alpha}t\right]\right] = (sF(s)-f(0))(\frac{1}{s+\frac{\alpha}{1-\alpha}})$$

It means we have:

$$\int_0^{\infty} \exp(-st)\int_0^t f'(x)\exp\left[\frac{-\alpha(t-x)}{1-\alpha}\right]dxdt = \big(sF(s) - f(0)\big)\left(\frac{1}{s+\frac{\alpha}{1-\alpha}}\right) \qquad (5)$$

Now equation (5) can be written as follows:

$$\frac{M(\alpha)}{1-\alpha}\left(sF(s)-f(0)\right)\left(\frac{1}{s+\frac{\alpha}{1-\alpha}}\right)=F(s) \qquad (6)$$

$$\frac{M(\alpha)}{1-\alpha}\left(sF(s)-f(0)\right)\left(\frac{1-\alpha}{s+\alpha(1-s)}\right)=F(s)$$

By simplifying the above equation, we have:

$$M(\alpha)(\frac{sF(s)-F(0)}{s+\alpha(1-s)}=F(s)$$

$$F(s)=\frac{f(0)-1}{\alpha(1-s)} \qquad (7)$$

By reversing the Laplace transform operator to the both side equation of (6)(7) relationship, we will have:

$$f(t)=\frac{M(\alpha)f(0)}{\alpha}\exp[t]$$

And this completes the proof.

Theorem 3.2: The **Caputo-Fabrizio derivative** is a fractional derivative, and f(t) is the function on which this derivative acts . The Sumudu transform is applied to this fractional derivative.

The corrected form should be:

$$S({}^{CF}_{0}D^{\alpha}_{t})f(t)(u)=M(\alpha)\frac{F(u)-f(0)}{1-\alpha+\alpha u}$$

Proof: According to the definition of Sumudu and the Caputo-Fabrizio derivation have:

$$S({}^{CF}_{0}D^{\alpha}_{t}(f(t))(u)=\frac{M(\alpha)}{1-\alpha}\int_0^\infty\frac{1}{u}\exp\left(\frac{-t}{u}\right)\int_0^t\frac{df(\tau)}{d\tau}\exp\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right]d\tau dt \qquad (8)$$

On the other hand,

$$\int_0^t\frac{df(\tau)}{d\tau}\exp\left[\frac{-\alpha}{1-\alpha}(t-x)dx\right. \qquad (9)$$

That's the same convolution of $\exp\left(\frac{-\alpha}{1-\alpha}(t)\right),\frac{df(t)}{dt}$ so, by applying the convolution feature to Sumudu transform, will have:

$S({}^{CF}_{0}D^{\alpha}_{t})f(t)(u) = \frac{M(\alpha)}{1-\alpha} S\frac{df(t)}{dt}.\exp\,[\frac{-\alpha}{1-\alpha}(t)])$,

$= \frac{M(\alpha)}{1-\alpha} u\, S(\frac{df(t)}{dt} S(\exp\left[\frac{-\alpha}{1-\alpha}(t)\right])$,

$= \frac{M(\alpha)}{1-\alpha} u(\frac{F(u)-f(0)}{u})(\frac{1}{1+\frac{\alpha}{1-\alpha}u})$,

$$= \frac{M(\alpha)}{1-\alpha} u\big(F(u)-f(0)\big)\left(\frac{1-\alpha}{1-\alpha+\alpha u}\right) , \qquad (10)$$

$$= M(\alpha)\frac{(F(u)-f(0))}{1-\alpha+\alpha u}$$

This completes the proof.

Corollary 3.2: For the desired natural number ℕ and the $n - times$ derivable function of we have:

$$S({}^{CF}_{0}D^{\alpha+n}_{t}\big(f(t)\big)(u) = \frac{M(\alpha)}{1-\alpha}\int_0^{\infty}\frac{1}{u}\exp\,(\frac{-t}{u})\int_0^{t}\frac{df^{n+1}(\tau)}{dt^{n+1}}\exp\,[\frac{-\alpha}{1-\alpha}(t-\tau)]d\tau dt$$

Proof. Considering the definition of the Sumudu transform, have:

$$S({}^{CF}_{0}D^{\alpha+n}_{t}\big(f(t)\big)(u) = \frac{M(\alpha)}{1-\alpha}\int_0^{\infty}\frac{1}{u}\exp\,(\frac{-t}{u})\int_0^{t}\frac{df^{n+1}(\tau)}{dt^{n+1}}\exp\,[\frac{-\alpha}{1-\alpha}(t-\tau)]d\tau dt$$

Can see that

$$\int_0^t \frac{df^{n+1}(\tau)}{dt^{n+1}}\exp\,[\frac{-\alpha}{1-\alpha}(t-\tau)]d\tau \qquad (11)$$

The convolutions are $\exp\left[\frac{-\alpha}{1-\alpha}(t)\right]$ $and\ f^{(n+1)}(t)$. So, will have:

$$S({}^{CF}_{0}D^{\alpha}_{t})f(t)(u) = \frac{M(\alpha)}{1-\alpha} S(\frac{df^{n+1}(t)}{dt^{n+1}} * \exp\left(\frac{-\alpha}{1-\alpha}\right)t) \qquad (12)$$

$$= \frac{M(\alpha)}{1-\alpha} uS(\frac{df^{n+1}(t)}{dt^{n+1}})S(\exp\left(\frac{-\alpha}{1-\alpha}\right)t) \qquad (13)$$

$$= \frac{M(\alpha)}{1-\alpha} u\{\frac{F(u)}{u^{n+1}} - \sum_{k=0}^{n}\frac{f^{(k)}(0)}{u^{n+1-k}}\}\ \frac{1}{1+\frac{\alpha}{1-\alpha}u} \qquad (14)$$

$$= M(\alpha)\left\{\frac{F(u)}{u^n} - \sum_{k=0}^{n} \frac{f^{(k)}(0)}{u^{n-k}}\right\}\frac{1}{1-\alpha+\alpha u} \quad (15)$$

And the proof is completed.

Theorem 3.3. For the desired natural number $1 \leq n$, the Caputo-Fabrizio fractional derivation of order $0 < \alpha < 1$ for t^n is defined as follows:

$${}_{0}^{CF}D_t^{\alpha}(t^n) = \frac{M(\alpha)}{\alpha}\sum_{i=0}^{n-1}(-1)^i \frac{n!\,x^{n-i-1}}{(n-i-1)}\left(\frac{1-\alpha}{\alpha}\right)^{i+1}$$

proof of a new fractional derivation for t^n is as follows:

$${}_{0}^{CF}D_t^{\alpha}(t^n) = \frac{M(\alpha)}{1-\alpha}\int_0^t \frac{d}{d\tau}(\tau^n)\exp\left(-\alpha\frac{t-\tau}{1-\alpha}\right)d\tau \quad (16)$$

$$\frac{M(\alpha)}{1-\alpha}\int_0^t n\tau^{n-1}\exp\left(-\alpha\frac{t-\tau}{1-\alpha}\right)d\tau \quad (17)$$

$$\frac{M(\alpha)}{1-\alpha}\exp\left(\frac{-\alpha\tau}{1-\alpha}\right)\int_0^t n\tau^{n-1}\exp\left(\frac{\alpha\tau}{1-\alpha}\right)d\tau \quad (18)$$

First, calculate the following integral:

$$\int_0^t n\tau^{n-1}\exp\left(\frac{\alpha\tau}{1-\alpha}\right)d\tau \quad (19)$$

For simplicity, assume:

$$c = \frac{\alpha}{1-\alpha} \quad (20)$$

Through part-by-part integral will have:

$$\int_0^t n\tau^{n-1}\exp(c\tau)\,d\tau = \frac{n\tau^{n-1}}{c}\exp(c\tau)\,|_0^t - \frac{1}{c}\int_0^t n(n-1)\tau^{n-2}\exp(c\tau)\,d\tau \quad (21)$$

$$= \frac{n}{c}t^{n-1}\exp(ct) - \frac{1}{c}\int_0^t n(n-1)\tau^{n-2}\exp(c\tau)\,d\tau \quad (22)$$

Similarly, have:

$$\frac{1}{c}\int_0^t n(n-1)\,\tau^{n-2}\exp(c\tau)\,d\tau = \frac{n(n-1)}{c^2}t^{n-2}\exp(t) - \frac{1}{c^2}\int_0^t n(n-1)(n-2)\tau^{n-3}\exp(c\tau)\,d\tau \quad (23)$$

By continuing this trend, will have:

$$\int_0^t n\tau^{n-1}\exp(c\tau)\,d\tau = \exp\,(c\tau)\left\{\sum_{i=0}^{n-1}(-1)^i\frac{n!}{(n-i-1)!}\frac{t^{n-i-1}}{c^{i+1}}\right\} \quad (24)$$

So, by replacing 23 in 24 in the definition of the fractional derivation, have:

$${}_0^{CF}D_t^{\alpha}(t^n) = \frac{M(\alpha)}{1-\alpha}\exp(-c\tau)\,exp(c\tau)\left\{\sum_{i=0}^{n-1}(-1)^i\frac{n!}{(n-i-1)!}\frac{t^{n-i-1}}{c^{i+1}}\right\} \quad (25)$$

$$\frac{M(\alpha)}{1-\alpha}\left\{\sum_{i=0}^{n-1}(-1)^i\frac{n!}{(n-i-1)!}\frac{t^{n-i-1}}{c^{i+1}}\right\} \quad (26)$$

Theorem 3.4: The Caputo-Fabrizio fractional derivation for the function $\ln(t)$ is as follows:

$${}_0^{CF}D_t^{\alpha}(\ln\,(t)) = \frac{M(\alpha)}{1-\alpha}\exp\left(\frac{-\alpha t}{1-\alpha}\right)\left[ln|t| + \sum_{n=0}^{\infty}\frac{\left(\frac{\alpha t}{1-\alpha}\right)^n}{n!\,n}\right]$$

proof. Considering the new derivation, have:

$${}_0^{CF}D_t^{\alpha}(\ln(t)) = \frac{M(\alpha)}{1-\alpha}\int_0^t \frac{d}{d\tau}(\ln(\tau))\exp\left(-\alpha\frac{t-\tau}{1-\alpha}\right)d\tau$$

$$= \frac{M(\alpha)}{1-\alpha}\exp\left(\frac{-\alpha t}{1-\alpha}\right)\int_0^t \frac{1}{\tau}\exp\left(\frac{\alpha\tau}{1-\alpha}\right)d\tau \quad (27)$$

If put $c = \frac{\alpha}{1-\alpha}$, the expansion of the MacLaurin function of $\exp\,(c\tau)$ is as follows:

$$\exp(c\tau) = \sum_{n=0}^{\infty}\frac{(c\tau)^n}{n!} \quad (28)$$

And considering it, have:

$$\int_0^t \frac{\exp\,(c\tau)}{\tau}d\tau = \int_0^t \frac{\sum_{n=0}^{\infty}\frac{(c\tau)^n}{n!}}{\tau}d\tau$$

$$= \int_0^t (\frac{1}{\tau} + c + \frac{c^2}{2!}\tau + \frac{c^3}{3!}\tau^2 + \frac{c^4}{4!}\tau^3 + \cdots)\tau$$

$$= (ln|t| + c\tau + \frac{c^2\tau^2}{2!2} + \frac{c^3\tau^3}{3!3} + \frac{c^4\tau^4}{4!4} + \cdots)|_0^t$$

$$= ln|t| + \sum_{n=0}^{\infty} \frac{(ct)^n}{n!n} \quad (29)$$

This completes the proof.

Theorem 3.5: For the desired β>1, the Caputo-Fabrizio fractional derivation for t^β is as follows:

$${}_0^{CF}D_t^\alpha\left(t^\beta\right) = \frac{-\beta M(\alpha)}{1-\alpha} \exp\left(-\frac{\alpha t}{1-\alpha}\right) t^\beta \left(-\frac{\alpha t}{1-\alpha}\right)^{-\beta} \gamma\left(\beta, -\frac{\alpha t}{1-\alpha}\right)$$

proof. Considering the definition of fractional derivation, have:

$${}_0^{CF}D_t^\alpha\left(t^\beta\right) = \frac{M(\alpha)}{1-\alpha} \int_0^t \frac{d}{d\tau}\left(\tau^\beta\right) \exp\left(-\alpha \frac{t-\tau}{1-\alpha}\right) d\tau$$

$$= \frac{M(\alpha)}{1-\alpha} \int_0^t \beta\tau^{\beta-1} \exp\left(-\alpha \frac{t-\tau}{1-\alpha}\right)$$

$$= \beta \frac{M(\alpha)}{1-\alpha} \exp\left(-\frac{\alpha t}{1-\alpha}\right) \int_0^t \tau^{\beta-1} \exp\left(-\alpha \frac{\tau}{1-\alpha}\right) d\tau \quad (30)$$

put $c = \frac{\alpha}{1-\alpha}$, so have:

$${}_0^{CF}D_t^\alpha\left(t^\beta\right) = \beta \frac{M(\alpha)}{1-\alpha} \exp(-ct) \int_0^t \tau^{\beta-1} \exp(-c\tau)\, d\tau \quad (31)$$

Through using the change of $c\tau = -u$ variable will have:

$$\int_0^t \tau^{\beta-1}\exp(-c\tau)\,d\tau = \int_0^{-ct}(-\frac{u}{c})^{\beta-1}\exp(-u)\left(-\frac{1}{u}\right)du \qquad (32)$$

But considering the previous chapter know:

$$= (-\frac{1}{c})^{\beta}\int_0^{-ct} u^{\beta-1}\exp(-u)\,du \qquad (33)$$

So,

$$\int_0^t \tau^{\beta-1}\exp(-c\tau)\,d\tau = \left(-\frac{1}{\frac{\alpha}{1-\alpha}}\right)^{\beta}\gamma(\beta,-\frac{\alpha t}{1-\alpha})$$

$$= \left(\frac{-\alpha}{1-\alpha}\right)^{-\beta}\gamma\left(\beta,-\frac{\alpha t}{1-\alpha}\right) \qquad (34)$$

By replacing 33 relationship with 34 will have:

$${}^{CF}_{0}D^{\alpha}_{t}\left(t^{\beta}\right) = \frac{\beta M(\alpha)}{1-\alpha}\exp\left(-\frac{\alpha t}{1-\alpha}\right)\left(-\frac{\alpha}{1-\alpha)}\right)^{-\beta}\gamma(\beta,-\frac{\alpha t}{1-\alpha)} \qquad (35)$$

3.6 Algorithm for Nonlinear Fisher Diffusion Equation:

Input:

$u_n(x,t) = u(x,0)$ (initial condition at $t = 0$ typically a given function like $\cos(x)$)

n (number of approximation steps)

m =6 (order of the nonlinear term)

β (a constant)

γ (parameter for fractional derivative)

s (a scaling factor)

M(γ) (a function or operator related to the fractional derivative)

Output:

approximate solution $u_{app}(x,t)$

Stages:

Stage 1: Initial Condition

Set the initial condition for $u_n = (x,t)$:

$$u_0(x,t) = u(x,0)$$

This is the initial value of the solution at $t = 0$, which is provided in the problem (e.g., $u(x,0)$=cos(x)). (36)

Set $u_{app}(x,t) = u_0(x,t)$ as the initial approximation.

Stage 2: Iterative Update (Time Stepping)

For $j = 1$ to $n - 1$, perform the following updates

fractional update of $u_n(x,t)$:

$$u_{n+1} = u_n(x,t) + s^{-1}\left\{\frac{1+\gamma-\gamma s}{M(\gamma)} S\left\{\alpha \frac{\vartheta^2\, u_n(x,t)}{\partial x^2} - \beta u_n(x,t)(1-u^m(x,t))\right\}\right\}$$

S represents some fractional integration or operator,

α is a constant scaling factor,

The term $\frac{\vartheta^2 u_n(x,t)}{\vartheta x^2}$ represents the second spatial derivative of $u_n(x,t)$, (1- $u_n^m(x,t)$) is the nonlinear reaction term, $M(\gamma)$ might be related to a normalization factor for the fractional derivative.

Stage 3: Update the Approximate Solution $u_{app}(x$, t)

$u_{\text{app}}(x,t) = u_n(x,t) + u_{app}(x,t)$

Stage 4: Final Update for Next Iteration

Set:

$u_{app}(x,t) = u_{n+1}(x,t) + u_{app}(x,t)$

This updates the approximate solution for the next step in the iteration. The above steps are repeated for n iterations to simulate the evolution of the solution $u(x,t)$ over time. the method handles fractional time derivatives and the nonlinear Fisher equation. The results show the evolution of $u(x,t)$ under varying fractional order values of γ, which affect the diffusion speed and behavior.

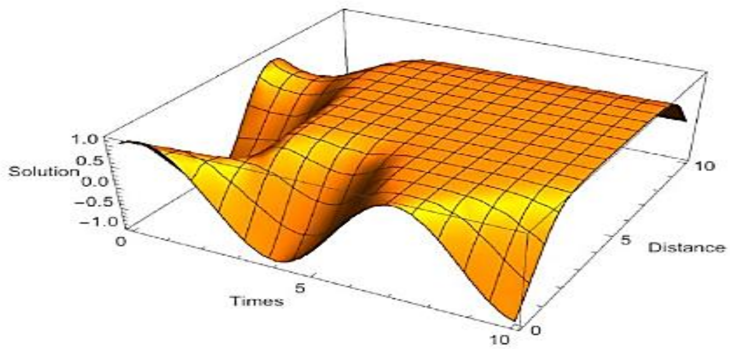

Figure 1: Numerical assimilation for (36) with m=6, β=1

In the above figure, the numerical simulation of equation (36) is done with $m = 6$ and $\beta = 1$. The following is a numerical repetition of equation (36) with the initial condition of $u(x,0) = cosx$,

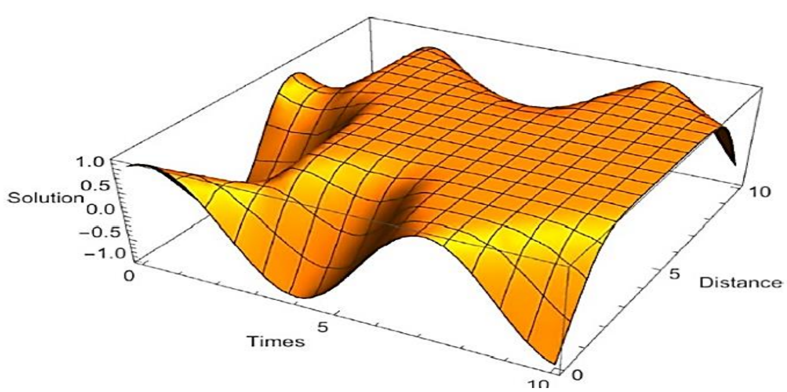

Figure 2: Numerical assimilation for (36) with $m = 6, \beta = 0.85$

The results obtained from graphical representation show that the proposed model is significantly dependent on the fractional order and there is a clear difference between the case where the fractional order is 1 and the case which is 0.85. When the fractional order is 0.85, this model shows a feature which in the case of the model with the integer this feature was hidden. The fractional order model presented in Figure 2 shows that the limiting wave fronts, as well as their diffusion velocity, do not depend on the initial values, but depending on the complexity of the environment that diffusion is done through, these complexities can be seen by changing the order of the fraction.

Conclusions

The fractional derivative proposed by Caputo and Fabrizio has some interesting properties to consider, among other things, it can describe heterogeneities and simple configurations with different scales that clearly cannot be supervised by popular topical theories A complementary application in learning the microscopic behavior of some materials is related to nonlocal connections between atoms that are known to be important properties of materials. With these features, have presented useful tools related to fractional order derivation. have used this new derivation to modify the Fisher's diffusion equation. obtained the private solution by applying the Sumudu transform related to the fractional Lagrange coefficient.

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معادلة فيشر غير الخطية للانتشار-التفاعل: اشتقاق كسري جديد والنظريات المرتبطة بها والمحاكاة العددية

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الخلاصة :

مؤخرًا، قدّم كابوتو-فابريزيو مشتقًا جديدًا من الدرجة الكسرية يتمتع بأدوات وتطبيقات قيّمة، وقد تم استكشافه في هذا العمل. تم تطبيق هذا المشتق الجديد على مشكلة فيشر غير الخطية للانتشار-التفاعل، وتم اشتقاق حل للمعادلة المعدلة باستخدام عملية تكرارية. تم إثبات استقرار النهج المقترح باستخدام نظرية النقطة الثابتة. كما تم تقديم محاكاة عددية لقيم مختلفة من الدرجة الكسرية، بالإضافة إلى تقديم نظريات هامة مع إثباتاتها. تحل معادلات الانتشار الكسرية محل المشتق الثاني المكاني بمشتق كسري من الدرجة أقل من اثنين، وهي مفيدة بشكل خاص في السيناريوهات التي تنتشر فيها سحب الجسيمات بشكل أسرع مما تتنبأ به النظرية الكلاسيكية. تظهر هذه الحلول عدم تناسق وتنتشر بسرعة أكبر من الحلول الكلاسيكية. تجمع معادلات الانتشار-التفاعل الكسرية بين الانتشار المركب ومصطلح التفاعل الكلاسيكي. يقترح دراستنا تقنية عددية عملية تعتمد على تقسيم المشغل لحل هذه المعادلات. يتم فحص خصائص الحلول العددية وتقديم محاكاة عددية للتحقق من صحة الطريقة. بالإضافة إلى ذلك، نناقش التطبيقات البيولوجية، حيث يمثل مصطلح التفاعل توسع الأنواع، بينما يعبر مصطلح الانتشار عن الحركة ..