13-Brauer Trees for Spin Characters of Symmetric Group S_{23}

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Abstract

In this paper 13-Brauer Trees for Spin Characters of S_{23} modulo $p = 13$ which gives the 13-Brauer characters for a spin characters of S_{23} are calculated.

Key words: spin characters, projective representation, Brauer trees.

1 Introduction

The symmetric groups S_n has a covering group $\overline{S_n}$ of order $2(n!)$ [1]. The projective (spin) representation of a group G is a homomorphism $G \longrightarrow PGL(V)$, where $PGL(V)$ denotes the projective general linear group of an $\mathbb{F}$-vector space V [2]. There are two types of a projective representations, first is when the field $\mathbb{F}$ is of characteristic zero which is called the ordinary projective representation and second is a modular projective representation that is when the characteristic of $\mathbb{F}$ is a prime number. There is no general method to calculate the decomposition matrix for an arbitrary group G [3]. In this paper Bruaer trees (decomposition matrix) for S_{23} modulo $p = 13$ has been calculated by using $(r, \bar{r})$-inducing method [4,5], and technique for finding decomposition matrices [3], Where the decomposition matrices for a spin characters for the symmetric groups S_{21} and S_{22} are found by [6,7], which is given in the appendix I. Most fact using in this paper can be found in [3,4,5,8].

2 13-Blocks for S_{23}

The symmetric group S_{23} has 43 blocks, ten of them $B_1, B_2, \dots, B_{10}$ are of defect one while the others are of defect zero [4], Where

$B_1 = B_1^1 \cup B_1^2,\quad B_1^1 = \{\langle 23\rangle^*, \langle 12,10,1\rangle^*, \langle 10,9,4\rangle^*, \langle 10,7,6\rangle^*\}$

$B_1^2 = \{\langle 13,10\rangle, \langle 13,10\rangle', \langle 11,10,2\rangle^*, \langle 10,8,5\rangle^*\}$

$B_2 = B_2^1 \cup B_2^2,\quad B_2^1 = \{\langle 22,1\rangle, \langle 22,1\rangle', \langle 13,9,1\rangle^*, \langle 10,9,3,1\rangle, \langle 10,9,3,1\rangle', \langle 9,7,6,1\rangle, \langle 9,7,6,1\rangle'\}$

$B_2^2 = \{\langle 14,9\rangle, \langle 14,9\rangle', \langle 11,9,2,1\rangle, \langle 11,9,2,1\rangle', \langle 9,8,5,1\rangle, \langle 9,8,5,1\rangle'\}$

$B_3 = B_3^1 \cup B_3^2,\quad B_3^1 = \{\langle 21,2\rangle, \langle 21,2\rangle', \langle 13,8,2\rangle^*, \langle 10,8,3,2\rangle, \langle 10,8,3,2\rangle', \langle 8,7,6,2\rangle, \langle 8,7,6,2\rangle'\}$

$B_3^2 = \{\langle 15,8\rangle, \langle 15,8\rangle', \langle 12,8,2,1\rangle, \langle 12,8,2,1\rangle', \langle 9,8,4,2\rangle, \langle 9,8,4,2\rangle'\}$

$B_4 = B_4^1 \cup B_4^2,\quad B_4^1 = \{\langle 20,3\rangle, \langle 20,3\rangle', \langle 13,7,3\rangle^*, \langle 11,7,3,2\rangle, \langle 11,7,3,2\rangle', \langle 8,7,5,3\rangle, \langle 8,7,5,3\rangle'\}$

$B_4^2 = \{\langle 16,7\rangle, \langle 16,7\rangle', \langle 12,7,3,1\rangle, \langle 12,7,3,1\rangle', \langle 9,7,4,3\rangle, \langle 9,7,4,3\rangle'\}$

$B_5 = B_5^1 \cup B_5^2,\quad B_5^1 = \{\langle 20,2,1\rangle^*, \langle 14,7,2\rangle^*, \langle 10,7,3,2,1\rangle^*, \langle 8,7,5,2,1\rangle^*\}$

$B_5^2 = \{\langle 15,7,1\rangle^*, \langle 9,7,4,2,1\rangle^*, \langle 13,7,2,1\rangle, \langle 13,7,2,1\rangle'\}$

$B_6 = B_6^1 \cup B_6^2,\quad B_6^1 = \{\langle 19,4\rangle, \langle 19,4\rangle', \langle 13,6,4\rangle^*, \langle 11,6,4,2\rangle, \langle 11,6,4,2\rangle', \langle 8,6,5,4\rangle, \langle 8,6,5,4\rangle'\}$

$B_6^2 = \{\langle 17,6\rangle, \langle 17,6\rangle', \langle 12,6,4,1\rangle, \langle 12,6,4,1\rangle', \langle 10,6,4,3\rangle, \langle 10,6,4,3\rangle'\}$

$B_7 = B_7^1 \cup B_7^2,\quad B_7^1 = \{\langle 19,3,1\rangle^*, \langle 14,6,3\rangle^*, \langle 11,6,3,2,1\rangle^*, \langle 8,6,5,3,1\rangle^*\}$

$B_7^2 = \{\langle 13,6,3,1\rangle, \langle 13,6,3,1\rangle', \langle 16,6,1\rangle^*, \langle 9,6,4,3,1\rangle^*\}$

$B_8 = B_8^1 \cup B_8^2,\quad B_8^1 = \{\langle 18,4,1\rangle^*, \langle 14,5,4\rangle^*, \langle 11,5,4,2,1\rangle^*, \langle 7,6,5,4,1\rangle^*\}$

$B_8^2 = \{\langle 17,5,1\rangle^*, \langle 10,5,4,3,1\rangle^*, \langle 13,5,4,1\rangle, \langle 13,5,4,1\rangle'\}$

$B_9 = B_9^1 \cup B_9^2,\quad B_9^1 = \{\langle 18,3,2\rangle^*, \langle 15,5,3\rangle^*, \langle 12,5,3,2,1\rangle^*, \langle 7,6,5,3,2\rangle^*\}$

$B_9^2 = \{\langle 16,5,2\rangle^*, \langle 9,5,4,3,2\rangle^*, \langle 13,5,3,2\rangle, \langle 13,5,3,2\rangle'\}$

$B_{10} = B_{10}^1 \cup B_{10}^2,$
$B_{10}^1 = \{\langle 17,3,2,1\rangle, \langle 17,3,2,1\rangle', \langle 15,4,3,1\rangle, \langle 15,4,3,1\rangle', \langle 13,4,3,2,1\rangle^*, \langle 7,6,4,3,2,1\rangle, \langle 7,6,4,3,2,1\rangle'\}$

$B_{10}^2 = \{\langle 16,4,2,1\rangle, \langle 16,4,2,1\rangle', \langle 14,4,3,2\rangle, \langle 14,4,3,2\rangle', \langle 8,5,4,3,2,1\rangle, \langle 8,5,4,3,2,1\rangle'\}$

3 13-Brauer Trees for Blocks of S_{23}

In this section, the Brauer trees for the symmetric group S_{23} modulo 13 are found. Throughout this section all 13-Brauer trees denoted by B.T.

Lemma 3.1 The B.T. for the principle block B_1 is

$$\langle 23\rangle^* - \langle 13,10\rangle = \langle 13,10\rangle' - \langle 12,10,1\rangle^* - \langle 11,10,2\rangle^* - \langle 10,9,4\rangle^* - \langle 10,8,5\rangle^* - \langle 10,7,6\rangle^*$$

Proof:

Since $\langle\lambda\rangle = \langle\lambda\rangle'$ on $(13,\alpha)$-regular classes in this block and $deg\langle\lambda\rangle \equiv 7 \ mod\ 13 \ \ \forall\, \lambda \in B_1^1$, $deg\langle\beta\rangle \equiv -7 \ mod\ 13 \ \ \forall\, \beta \in B_1^2$ and by $(10,4)$-inducing of a principal indecomposable spin characters for S_{22} (given in appendix I) to S_{23} we have

$d_1 \uparrow^{(10,4)} = \langle 23\rangle^* + \langle 13,10\rangle + \langle 13,10\rangle' = D_1$

$d_3 \uparrow^{(10,4)} = \langle 13,10\rangle + \langle 13,10\rangle' + \langle 12,10,1\rangle^* = D_2$

$d_5 \uparrow^{(10,4)} = \langle 12,10,1\rangle^* + \langle 11,10,2\rangle^* = D_3$

$d_7 \uparrow^{(10,4)} = \langle 11,10,2\rangle^* + \langle 10,9,4\rangle^* = D_4$

$d_9 \uparrow^{(10,4)} = \langle 10,9,4\rangle^* + \langle 10,8,5\rangle^* = D_5$

$d_{11} \uparrow^{(10,4)} = \langle 10,8,5\rangle^* + \langle 10,7,6\rangle^* = D_6$

Then we have the Brauer tree for B_1. ■

Lemma 3.2 The B.T of the block B_2 is

$$
\begin{array}{ccccc}
\langle 22,1\rangle - \langle 14,9\rangle & & & & \langle 11,9,2,1\rangle - \langle 10,9,3,1\rangle - \langle 9,8,5,1\rangle - \langle 9,7,6,1\rangle \\
 & \diagdown & & \diagup & \\
 & & \langle 13,9,1\rangle^* & & \\
 & \diagup & & \diagdown & \\
\langle 22,1\rangle' - \langle 14,9\rangle' & & & & \langle 11,9,2,1\rangle' - \langle 10,9,3,1\rangle' - \langle 9,8,5,1\rangle' - \langle 9,7,6,1\rangle'
\end{array}
$$

Proof:

Using the (9,5)-inducing of $d_{12}, d_{13}, d_{14}, d_{15}, d_{16}, d_{17}$ to S_{23} we have

$d_{12} \uparrow^{(9,5)} = \langle 22,1\rangle + \langle 22,1\rangle' + \langle 14,9\rangle + \langle 14,9\rangle' = h_1$

$d_{13} \uparrow^{(9,5)} = \langle 14,9\rangle + \langle 14,9\rangle' + 2\langle 13,9,1\rangle^* = h_2$

$d_{14} \uparrow^{(9,5)} = 2\langle 13,9,1\rangle^* + \langle 11,9,2,1\rangle + \langle 11,9,2,1\rangle' = h_3$

$d_{15} \uparrow^{(9,5)} = \langle 11,9,2,1\rangle + \langle 11,9,2,1\rangle' + \langle 10,9,3,1\rangle + \langle 10,9,3,1\rangle' = h_4$

$d_{16} \uparrow^{(9,5)} = \langle 10,9,3,1\rangle + \langle 10,9,3,1\rangle' + \langle 9,8,5,1\rangle + \langle 9,8,5,1\rangle' = h_5$

$d_{17} \uparrow^{(9,5)} = \langle 9,8,5,1\rangle + \langle 9,8,5,1\rangle' + \langle 9,7,6,1\rangle + \langle 9,7,6,1\rangle' = h_6$

Since $\langle 22,1\rangle \neq \langle 22,1\rangle'$ and $\langle 22,1\rangle$is irreducible[9], then h_1 splits to give $\langle 22,1\rangle + \langle 14,9\rangle$ and $\langle 22,1\rangle' + \langle 14,9\rangle'$. In block B_2 we have $\langle \lambda\rangle \neq \langle \lambda\rangle'$ on (13,α)-regular classes thus $h_2, h_3, \dots, h_6$ split, by using [4], so we have the B.T. ■

Lemma 3.3 The Brauer tree of the block B_3 is

$$
\begin{array}{ccccc}
\langle 21,2\rangle - \langle 15,8\rangle & & & & \langle 12,8,2,1\rangle - \langle 10,8,3,2\rangle - \langle 9,8,4,2\rangle - \langle 8,7,6,2\rangle \\
 & \diagdown & & \diagup & \\
 & & \langle 13,8,2\rangle^* & & \\
 & \diagup & & \diagdown & \\
\langle 21,2\rangle' - \langle 15,8\rangle' & & & & \langle 12,8,2,1\rangle' - \langle 10,8,3,2\rangle' - \langle 9,8,4,2\rangle' - \langle 8,7,6,2\rangle'
\end{array}
$$

Proof:

By (8,6)-inducing of $d_{19}, \dots, d_{24}$ we have

$d_{19} \uparrow^{(8,6)} = \langle 21,2\rangle + \langle 21,2\rangle' + \langle 15,8\rangle + \langle 15,8\rangle'$

$d_{20} \uparrow^{(8,6)} = \langle 15,8\rangle + \langle 15,8\rangle' + 2\langle 13,8,2\rangle^*$

$d_{21}\uparrow^{(8,6)}= 2\langle 13,8,2\rangle^{*} + \langle 12,8,2,1\rangle + \langle 12,8,2,1\rangle'$

$d_{22}\uparrow^{(8,6)}= \langle 12,8,2,1\rangle + \langle 12,8,2,1\rangle' + \langle 10,8,3,2\rangle + \langle 10,8,3,2\rangle'$

$d_{23}\uparrow^{(8,6)}= \langle 10,8,3,2\rangle + \langle 10,8,3,2\rangle' + \langle 9,8,4,2\rangle, \langle 9,8,4,2\rangle'$

$d_{24}\uparrow^{(8,6)}= \langle 9,8,4,2\rangle + \langle 9,8,4,2\rangle' + \langle 8,7,6,2\rangle + \langle 8,7,6,2\rangle'$

Since $\langle\lambda\rangle \neq \langle\lambda\rangle'$ on $(13,\alpha)$-regular classes, then all the principal characters are splits to give the B.T. ■

Lemma 3.4 The Brauer tree of the block B_4 is

$\langle 20,3\rangle - \langle 16,7\rangle$ ⟍ $\langle 13,7,3\rangle^{*}$ ⟋ $\langle 12,7,3,1\rangle - \langle 11,7,3,2\rangle - \langle 9,7,4,3\rangle - \langle 8,7,5,3\rangle$

$\langle 20,3\rangle' - \langle 16,7\rangle'$ ⟋ $\langle 13,7,3\rangle^{*}$ ⟍ $\langle 12,7,3,1\rangle' - \langle 11,7,3,2\rangle' - \langle 9,7,4,3\rangle' - \langle 8,7,5,3\rangle'$

Proof:

The (7,7)-inducing of $d_{25},\ldots,d_{30}$ gives

$d_{25}\uparrow^{(7,7)}= \langle 20,3\rangle + \langle 20,3\rangle' + \langle 16,7\rangle + \langle 16,7\rangle'$

$d_{26}\uparrow^{(7,7)}= \langle 16,7\rangle + \langle 16,7\rangle' + 2\langle 13,7,3\rangle^{*}$

$d_{27}\uparrow^{(7,7)}= 2\langle 13,7,3\rangle^{*} + \langle 12,7,3,1\rangle + \langle 12,7,3,1\rangle'$

$d_{28}\uparrow^{(7,7)}= \langle 12,7,3,1\rangle + \langle 12,7,3,1\rangle' + \langle 11,7,3,2\rangle + \langle 11,7,3,2\rangle'$

$d_{29}\uparrow^{(7,7)}= \langle 11,7,3,2\rangle + \langle 11,7,3,2\rangle' + \langle 9,7,4,3\rangle + \langle 9,7,4,3\rangle'$

$d_{30}\uparrow^{(7,7)}= \langle 9,7,4,3\rangle + \langle 9,7,4,3\rangle' + \langle 8,7,5,3\rangle + \langle 8,7,5,3\rangle'$

On $(13,\alpha)$-regular classes we have $\langle\lambda\rangle \neq \langle\lambda\rangle'$ in the block B_4 then all the principal characters are splits to give the Brauer tree. ■

Lemma 3.5 The Brauer tree of the block B_5 is

$\langle 20,2,1\rangle^{*} - \langle 15,7,1\rangle^{*} - \langle 14,7,2\rangle^{*} - \langle 13,7,2,1\rangle = \langle 13,7,2,1\rangle' - \langle 10,7,3,2,1\rangle^{*} - \langle 9,7,4,2,1\rangle^{*} - \langle 8,7,5,2,1\rangle^{*}$

Proof:

$$\deg\{\langle 13,7,2,1\rangle + \langle 13,7,2,1\rangle', \langle 15,7,1\rangle^{*}, \langle 9,7,4,2,1\rangle^{*}\} \equiv 7 \ mod\ 13$$

$$\deg\{\langle 20,2,1\rangle^{*}, \langle 14,7,2\rangle^{*}, \langle 10,7,3,2,1\rangle^{*}, \langle 8,7,5,2,1\rangle^{*}\} \equiv -7 \ mod\ 13$$

By (7,7)-inducing of $d_{31}, d_{33}, d_{35}, d_{37}, d_{39}, d_{41}$ to S_{23} since $\langle 13,7,2,1\rangle = \langle 13,7,2,1\rangle'$ on (13,α)-regular classes and $deg\langle\gamma\rangle \equiv 7\ mod\ 13\ \forall\ \gamma \in B_5^2$, $deg\langle\gamma\rangle \equiv -7\ mod\ 13\ \forall\ \gamma \in B_5^1$, then we have B.T. for this block.

Lemma 3.6 The block B_6 has B.T.

$\langle 19,3\rangle - \langle 17,6\rangle$ \ / $\langle 12,6,4,1\rangle - \langle 11,6,4,2\rangle - \langle 10,6,4,3\rangle - \langle 8,6,5,4\rangle$

$\langle 13,6,4\rangle^{*}$

$\langle 19,3\rangle' - \langle 17,6\rangle'$ / \ $\langle 12,6,4,1\rangle' - \langle 11,6,4,2\rangle' - \langle 10,6,4,3\rangle' - \langle 8,6,5,4\rangle'$

Proof:

Using (6,8)-inducing of $d_{43}, \dots, d_{48}$ to S_{23} we have

$d_{43} \uparrow^{(6,8)} S_{23} = \langle 19,4\rangle + \langle 19,4\rangle' + \langle 17,6\rangle + \langle 17,6\rangle'$

$d_{44} \uparrow^{(6,8)} S_{23} = \langle 17,6\rangle + \langle 17,6\rangle' + 2\langle 13,6,4\rangle^{*}$

$d_{45} \uparrow^{(6,8)} S_{23} = 2\langle 13,6,4\rangle^{*} + \langle 12,6,4,1\rangle + \langle 12,6,4,1\rangle'$

$d_{46} \uparrow^{(6,8)} S_{23} = \langle 12,6,4,1\rangle + \langle 12,6,4,1\rangle' + \langle 11,6,4,2\rangle + \langle 11,6,4,2\rangle'$

$d_{47} \uparrow^{(6,8)} S_{23} = \langle 11,6,4,2\rangle + \langle 11,6,4,2\rangle' + \langle 10,6,4,3\rangle + \langle 10,6,4,3\rangle'$

$d_{48} \uparrow^{(6,8)} S_{23} = \langle 10,6,4,3\rangle + \langle 10,6,4,3\rangle' + \langle 8,6,5,4\rangle + \langle 8,6,5,4\rangle'$

Since $\langle\lambda\rangle \neq \langle\lambda\rangle'$ on $(13,\alpha)$-regular classes, then all the principal characters are splits to give the Brauer tree. ■

Lemma 3.7 B.T. of the block B_7 is

$\langle 19,3,1\rangle^{*} - \langle 16,6,1\rangle^{*} - \langle 14,6,3\rangle^{*} - \langle 13,6,3,1\rangle = \langle 13,6,3,1\rangle' - \langle 11,6,3,2,1\rangle^{*} - \langle 9,6,4,3,1\rangle^{*} - \langle 8,6,5,3,1\rangle^{*}$

Proof:

Since $\langle\lambda\rangle = \langle\lambda\rangle'$ on $(13,\alpha)$-regular classes in this block and $deg\langle\lambda\rangle \equiv 7\ mod\ 13\ \ \forall\ \lambda \in B_1^1$,

$deg\langle\beta\rangle \equiv -7\ mod\ 13\ \ \forall\ \beta \in B_1^2$ and by (6,8)-inducing of $d_{49}, d_{51}, d_{53}, d_{55}, d_{57}, d_{59}$ to S_{23} then we have B.T. for the block B_7. ■

Lemma 3.8 The Brauer tree of the block B_8 is

$\langle 18,4,1\rangle^* - \langle 17,5,1\rangle^* - \langle 14,5,4\rangle^* - \langle 13,5,4,1\rangle = \langle 13,5,4,1\rangle' - \langle 11,5,4,2,1\rangle^* - \langle 10,5,4,3,1\rangle^* - \langle 7,6,5,4,1\rangle^*$

Proof:

On $(13,\alpha)$-regular classes we have $\langle 13,5,4,1\rangle = \langle 13,5,4,1\rangle'$ in this block B_1 and since $deg\langle\lambda\rangle \equiv -12\ mod\ 13\ \ \forall\ \lambda \in B_8^1$, $deg\langle\lambda\rangle \equiv 12\ mod\ 13\ \ \forall\ \lambda \in B_8^2$ and by (4,10)-inducing of a spin characters $d_{49}, d_{51}, d_{53}, d_{55}, d_{57}, d_{59}$ to S_{23} thus we have B.T. for B_8. ■

Lemma 3.9 The Brauer tree for B_9 is

$\langle 18,3,2\rangle^* - \langle 16,5,2\rangle^* - \langle 15,5,3\rangle^* - \langle 13,5,3,2\rangle = \langle 13,5,3,2\rangle' - \langle 12,5,3,2,1\rangle^* - \langle 9,5,4,3,2\rangle^* - \langle 7,6,5,3,2\rangle^*$

Proof:

By (5,9)-inducing of a spin characters $d_{61}, d_{63}, d_{65}, d_{67}, d_{69}, d_{71}$ to S_{23} since $deg\lambda \equiv 7\ mod\ 13\ \ \forall\ \lambda \in B_9^1$, $deg\lambda \equiv -7\ \ mod\ 13\ \forall\ \lambda \in B_9^2$ and $\lambda = \lambda'$on $(13,\alpha)$-regular classes in the block B_9, then we got the B.T. for B_9. ■

Theorem The decomposition matrix $D_{23,13}$ for the symmetric group S_{23} given in appendix II.

Proof:

Since all blocks for the symmetric group S_{23} (except the block B_{10}) are determined in the above lemmas, then we have only to determine the block B_{10}. Now since $deg\lambda \equiv 8\ mod\ 13\ \forall\ \lambda \in B_{10}^1$ and $deg\lambda \equiv -8\ mod\ 13\ \forall\ \lambda \in B_{10}^2$, by (1,0)-inducing of the spin characters $d_{61}, d_{62}, \dots, d_{66}, d_{68}, d_{69}, \dots, d_{72}$ for S_{22} to S_{23} we have the approximation matrix below (Table(1))

Table(1)

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}
$\langle 17,3,2,1\rangle$	1											
$\langle 17,3,2,1\rangle'$		1										
$\langle 16,4,2,1\rangle$	1		1									
$\langle 16,4,2,1\rangle'$		1		1								
$\langle 15,4,3,1\rangle$			1		1							
$\langle 15,4,3,1\rangle'$				1		1						
$\langle 14,4,3,2\rangle$					1	1	1					
$\langle 14,4,3,2\rangle'$					1	1		1				
$\langle 13,4,3,2,1\rangle^*$					1	1	1	1	1	1		
$\langle 8,5,4,3,2,1\rangle$									1		1	
$\langle 8,5,4,3,2,1\rangle'$										1		1
$\langle 7,6,4,3,2,1\rangle$											1	
$\langle 7,6,4,3,2,1\rangle'$												1
	L_{55}	L_{56}	L_{57}	L_{58}	L_{59}	L_{60}	L_{61}	L_{62}	L_{63}	L_{64}	L_{65}	L_{66}

Since $\langle 14,4,3,2,1\rangle$ and $\langle 14,4,3,2,1\rangle'$ are of defect zero in S_{24} then

$\langle 14,4,3,2,1\rangle \downarrow S_{23} = \langle 14,4,3,2\rangle + \langle 13,4,3,2,1\rangle^{*}$

$\langle 14,4,3,2,1\rangle' \downarrow S_{23} = \langle 14,4,3,2\rangle' + \langle 13,4,3,2,1\rangle^{*}$

So $d_{67} \uparrow^{(1,0)} S_{23} = L$ splits to $L_{61} + L_{62}$ which are indecomposable.

Either L_{59}, L_{60} indecomposable or $L_{59} - L_{62}$, $L_{60} - L_{61}$ indecomposable[3]. $L_{59} \equiv 0\ mod\ 13$ and $L_{59} - k_2 \equiv 0\ mod\ 13$ if L_{59} is indecomposable, then $\langle 14,4,3,2\rangle = \theta_5 + \theta_6 + \theta_7$ and $\langle 13,4,3,2,1\rangle^{*} = \theta_5 + \theta_6 + \theta_7 + \theta_8 + \theta_9 + \theta_{10}$ thus $\theta = \langle 13,4,3,2,1\rangle^{*} - \langle 14,4,3,2\rangle$ is modular.

Now, $\theta \downarrow S_{23} = \langle 12,4,3,2,1\rangle + \langle 12,4,3,2,1\rangle' + \langle 13,4,3,2\rangle^{*} - \langle 13,4,3,2\rangle^{*} - \langle 14,4,3,1\rangle$

$= \langle 12,4,3,2,1\rangle + \langle 12,4,3,2,1\rangle' - \langle 14,4,3,1\rangle$

which is not modular for S_{22} so L_{59} is not indecomposable. Hence $L_{62} \subset L_{59}$ and $L_{61} \subset L_{60}$ and we have the B.T. for B_{10}. ■

Appendix I

The decomposition matrix for S_{22}

Spin characters	$D_{22,13}$ for the block B_1											
⟨22⟩	1											
⟨22⟩′		1										
⟨13,9⟩*	1	1	1	1								
⟨12,9,1⟩			1		1							
⟨12,9,1⟩′				1		1						
⟨11,9,2⟩					1		1					
⟨11,9,2⟩′						1		1				
⟨10,9,3⟩							1		1			
⟨10,9,3⟩′								1		1		
⟨9,8,5⟩									1		1	
⟨9,8,5⟩′										1		1
⟨9,7,6⟩											1	
⟨9,7,6⟩′												1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}

Spin characters	$D_{22,13}$ for the block B_2					
⟨21,1⟩*	1					
⟨14,8⟩*	1	1				
⟨13,8,1⟩		1	1			
⟨13,8,1⟩′		1	1			
⟨11,8,2,1⟩*			1	1		
⟨10,8,3,1⟩*				1	1	
⟨9,8,4,1⟩*					1	1
⟨8,7,6,1⟩*						1
	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}

Spin characters	$D_{22,13}$ for the block B_3					
$\langle 20,2\rangle^*$	1					
$\langle 15,7\rangle^*$	1	1				
$\langle 13,7,2\rangle$		1	1			
$\langle 13,7,2\rangle'$		1	1			
$\langle 12,7,2,1\rangle^*$			1	1		
$\langle 10,7,3,2\rangle^*$				1	1	
$\langle 9,7,4,2\rangle^*$					1	1
$\langle 8,7,5,2\rangle^*$						1
	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}

Spin characters	$D_{22,13}$ for the block B_4					
$\langle 19,3\rangle^*$	1					
$\langle 16,6\rangle^*$	1	1				
$\langle 13,6,3\rangle$		1	1			
$\langle 13,6,3\rangle'$		1	1			
$\langle 12,6,3,1\rangle^*$			1	1		
$\langle 11,6,3,2\rangle^*$				1	1	
$\langle 9,6,4,3\rangle^*$					1	1
$\langle 8,6,5,3\rangle^*$						1
	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

Spin characters	$D_{22,13}$ for the block B_5											
$\langle 19,2,1\rangle$	1											
$\langle 19,2,1\rangle'$		1										
$\langle 15,6,1\rangle$	1		1									
$\langle 15,6,1\rangle'$		1		1								
$\langle 14,6,2\rangle$			1		1							
$\langle 14,6,2\rangle'$				1		1						
$\langle 13,6,2,1\rangle^*$					1	1	1	1				
$\langle 10,6,3,2,1\rangle$							1		1			
$\langle 10,6,3,2,1\rangle'$								1		1		
$\langle 9,6,4,2,1\rangle$									1		1	
$\langle 9,6,4,2,1\rangle'$										1		1
$\langle 8,6,5,2,1\rangle$											1	
$\langle 8,6,5,2,1\rangle'$												1
	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}

Spin characters	$D_{22,13}$ for the block B_6					
$\langle 18,4\rangle^*$	1					
$\langle 17,5\rangle^*$	1	1				
$\langle 13,5,4\rangle$		1	1			
$\langle 13,5,4\rangle'$		1	1			
$\langle 12,5,4,1\rangle^*$			1	1		
$\langle 11,5,4,2\rangle^*$				1	1	
$\langle 10,5,4,3\rangle^*$					1	1
$\langle 7,6,5,4\rangle^*$						1
	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}

Spin characters	$D_{22,13}$ for the block B7											
⟨18,3,1⟩	1											
⟨18,3,1⟩′		1										
⟨16,5,1⟩	1		1									
⟨16,5,1⟩′		1		1								
⟨14,5,3⟩			1		1							
⟨14,5,3⟩′				1		1						
⟨13,5,3,1⟩*					1	1	1	1				
⟨11,5,3,2,1⟩							1		1			
⟨11,5,3,2,1⟩′								1		1		
⟨9,5,4,3,1⟩									1		1	
⟨9,5,4,3,1⟩′										1		1
⟨7,6,5,3,1⟩											1	
⟨7,6,5,3,1⟩′												1
	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}	d_{55}	d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

Spin characters	$D_{22,13}$ for the block B8											
⟨17,3,2⟩	1											
⟨17,3,2⟩′		1										
⟨16,4,2⟩	1		1									
⟨16,4,2⟩′		1		1								
⟨15,4,3⟩			1		1							
⟨15,4,3⟩′				1		1						
⟨13,4,3,2⟩*					1	1	1	1				
⟨12,4,3,2,1⟩							1		1			
⟨12,4,3,2,1⟩′								1		1		
⟨8,5,4,3,2⟩									1		1	
⟨8,5,4,3,2⟩′										1		1
⟨7,6,4,3,2⟩											1	
⟨7,6,4,3,2⟩′												1
	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	d_{66}	d_{67}	d_{68}	d_{69}	d_{70}	d_{71}	d_{72}

Appendix II

The decomposition matrix for S_{23}

Spin characters	$D_{23,13}$ for the block B_1					
$\langle 23\rangle^*$	1					
$\langle 13,10\rangle$	1	1				
$\langle 13,10\rangle'$	1	1				
$\langle 12,10,1\rangle^*$		1	1			
$\langle 11,10,2\rangle^*$			1	1		
$\langle 10,9,4\rangle^*$				1	1	
$\langle 10,8,5\rangle^*$					1	1
$\langle 10,7,6\rangle^*$						1
	D_1	D_2	D_3	D_4	D_5	D_6

Spin characters	$D_{23,13}$ for the block B_2											
$\langle 22,1\rangle$	1											
$\langle 22,1\rangle'$		1										
$\langle 14,9\rangle$	1		1									
$\langle 14,9\rangle'$		1		1								
$\langle 13,9,1\rangle^*$			1	1	1	1						
$\langle 11,9,2,1\rangle$					1		1					
$\langle 11,9,2,1\rangle'$						1		1				
$\langle 10,9,3,1\rangle$							1		1			
$\langle 10,9,3,1\rangle'$								1		1		
$\langle 9,8,5,1\rangle$									1		1	
$\langle 9,8,5,1\rangle'$										1		1
$\langle 9,7,6,1\rangle$											1	
$\langle 9,7,6,1\rangle'$												1
	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}	D_{16}	D_{17}	D_{18}

Spin characters	$D_{23,13}$ for the block B3											
⟨21,2⟩	1											
⟨21,2⟩′		1										
⟨15,8⟩	1		1									
⟨15,8⟩′		1		1								
⟨13,8,2⟩*			1	1	1	1						
⟨12,8,2,1⟩					1		1					
⟨12,8,2,1⟩′						1		1				
⟨10,8,3,2⟩							1		1			
⟨10,8,3,2⟩′								1		1		
⟨9,8,4,2⟩									1		1	
⟨9,8,4,2⟩′										1		1
⟨8,7,6,2⟩											1	
⟨8,7,6,2⟩′												1
	D_{19}	D_{20}	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}

Spin characters	$D_{23,13}$ for the block B4											
⟨20,3⟩	1											
⟨20,3⟩′		1										
⟨16,7⟩	1		1									
⟨16,7⟩′		1		1								
⟨13,7,3⟩*			1	1	1	1						
⟨12,7,3,1⟩					1		1					
⟨12,7,3,1⟩′						1		1				
⟨11,7,3,2⟩							1		1			
⟨11,7,3,2⟩′								1		1		
⟨9,7,4,3⟩									1		1	
⟨9,7,4,3⟩′										1		1
⟨8,7,5,3⟩											1	
⟨8,7,5,3⟩′												1
	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}	D_{41}	D_{42}

Spin characters	$D_{23,13}$ for the block B_5					
$\langle 20,2,1\rangle^*$	1					
$\langle 15,7,1\rangle^*$	1	1				
$\langle 14,7,2\rangle^*$		1	1			
$\langle 13,7,2,1\rangle$			1	1		
$\langle 13,7,2,1\rangle'$			1	1		
$\langle 10,7,3,2,1\rangle^*$				1	1	
$\langle 9,7,4,2,1\rangle^*$					1	1
$\langle 8,7,5,2,1\rangle^*$						1
	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}

Spin characters	$D_{23,13}$ for the block B_6											
$\langle 19,4\rangle$	1											
$\langle 19,4\rangle'$		1										
$\langle 17,6\rangle$	1		1									
$\langle 17,6\rangle'$		1		1								
$\langle 13,6,4\rangle^*$			1	1	1	1						
$\langle 12,6,4,1\rangle$					1		1					
$\langle 12,6,4,1\rangle'$						1		1				
$\langle 11,6,4,2\rangle$							1		1			
$\langle 11,6,4,2\rangle'$								1		1		
$\langle 10,6,4,3\rangle$									1		1	
$\langle 10,6,4,3\rangle'$										1		1
$\langle 8,6,5,4\rangle$											1	
$\langle 8,6,5,4\rangle'$												1
	D_{49}	D_{50}	D_{51}	D_{52}	D_{53}	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}

Spin characters	$D_{23,13}$ for the block B7					
$\langle 19,3,1\rangle^*$	1					
$\langle 16,6,1\rangle^*$	1	1				
$\langle 14,6,3\rangle^*$		1	1			
$\langle 13,6,3,1\rangle$			1	1		
$\langle 13,6,3,1\rangle'$			1	1		
$\langle 11,6,3,2,1\rangle^*$				1	1	
$\langle 9,6,4,3,1\rangle^*$					1	1
$\langle 8,6,5,3,1\rangle^*$						1
	D_{61}	D_{62}	D_{63}	D_{64}	D_{65}	D_{66}

Spin characters	$D_{23,13}$ for the block B8					
$\langle 18,4,1\rangle^*$	1					
$\langle 17,5,1\rangle^*$	1	1				
$\langle 14,5,4\rangle^*$		1	1			
$\langle 13,5,4,1\rangle$			1	1		
$\langle 13,5,4,1\rangle'$			1	1		
$\langle 11,5,4,2,1\rangle^*$				1	1	
$\langle 10,5,4,3,1\rangle^*$					1	1
$\langle 7,6,5,4,1\rangle^*$						1
	D_{67}	D_{68}	D_{69}	D_{70}	D_{71}	D_{72}

Spin characters	$D_{23,13}$ for the block B9					
$\langle 18,3,2\rangle^*$	1					
$\langle 16,5,2\rangle^*$	1	1				
$\langle 15,5,3\rangle^*$		1	1			
$\langle 13,5,3,2\rangle$			1	1		
$\langle 13,5,3,2\rangle'$			1	1		
$\langle 12,5,3,2,1\rangle^*$				1	1	
$\langle 9,5,4,3,2\rangle^*$					1	1
$\langle 7,6,5,3,2\rangle^*$						1
	D_{73}	D_{74}	D_{75}	D_{76}	D_{77}	D_{78}

Spin characters	$D_{23,13}$ for the block B_{10}											
$\langle 17,3,2,1\rangle$	1											
$\langle 17,3,2,1\rangle'$		1										
$\langle 16,4,2,1\rangle$	1		1									
$\langle 16,4,2,1\rangle'$		1		1								
$\langle 15,4,3,1\rangle$			1		1							
$\langle 15,4,3,1\rangle'$				1		1						
$\langle 14,4,3,2\rangle$					1		1					
$\langle 14,4,3,2\rangle'$						1		1				
$\langle 13,4,3,2,1\rangle^{*}$							1	1	1	1		
$\langle 8,5,4,3,2,1\rangle$									1		1	
$\langle 8,5,4,3,2,1\rangle'$										1		1
$\langle 7,6,4,3,2,1\rangle$											1	
$\langle 7,6,4,3,2,1\rangle'$												1
	D_{79}	D_{80}	D_{81}	D_{82}	D_{83}	D_{84}	D_{85}	D_{86}	D_{87}	D_{88}	D_{89}	D_{90}

References

[1] J. Schur, Über die Darstellung der symmetrischen und der alternierenden Gruppe durch gebrochene lineare subtituttionen, Journal fur die reine und angewandte Math. 139 (1911) 155-250.

[2] C.W. Curtis and I. Reiner, Representation theory of finite groups and associative algebras, Sec. Printing, (1966).

[3] G. James and A. Kerber, The representation theory of the symmetric groups, Reading, Mass, Aaddiso-Wesley, (1981).

[4] A. K. Yaseen, Modular spin representations of the symmetric groups, Ph. D thesis, Aberystwyth (1987).

[5] J.F. Humphreys, Projective modular representations of finite groups I, J. London math. Society (2) 16 (1977) 51-66.

[6] A. K. Yaseen, The Brauer trees of the symmetric group S_{21} modulo p=13, Basra J. sci. 37 (1) 126-140 (2019).

[7] A. K. Yaseen, The Brauer trees of the symmetric group S_{22} modulo p=13, Manuscript.

[8] B. M. Puttaswamaiah, J. D. Dixon, Modular representation of finite groups, Academic press, (1977).

[9] D. B. Wales, Some projective representations of Sn, J. Algebra 61 (1979) 37-57.

13- اشجار براور من النمط 13 للزمرة التناظرية. S_{23}

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المستخلص

في هذا البحث تم حساب اشجار براور قياس $p = 13$ للمشخصات الاسقاطية للزمرة التناظرية S_{23} والتي تعطينا المشخصات الاسقاطية المعيارية قياس $p = 13$ للزمرة التناظرية S_{23} .