The Brauer trees of the symmetric group S_{21} modulo P=13

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Abstract

The main object of this paper is to find the Brauer Trees of the symmetric group S_{21} modulo $P=13$ which can give the irreducible modular spin characters of S_{21} Modulo $P=13$

Key words: Modular representations and characters, Brauer trees, decomposition matrix for the spin characters

1. Introduction

The Symmetric group S_n has a representation group $\overline{S_n}$ of order 2($n!$) , then the irreducible representations or characters of $\overline{S_n}$ are of the two distinct types [1, 2].

1. The representation of S_n which is called the ordinary representation.The irreducible representations and characters corresponding to partition λ of n denoted by $(\lambda)=(\lambda_1,\lambda_2,.......,\lambda_m)$ and $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_m$ >0

2. The representation of S_n which is called the spin representation. The irreducible representations are indexed by partitions λ of distinct parts which are called bar partitions written $\lambda \mapsto n$ [2, 3].

In fact, if $(\lambda)=(\lambda_1,\lambda_2,.......,\lambda_n)$, $\lambda \mapsto n$ and if $n-m$ is even, then there is one irreducible spin character denoted by $\langle\lambda\rangle$ which is self- associate and if $n-m$ is odd, then there are two associate spin characters denoted by $\langle\lambda\rangle$ and $\langle\lambda\rangle'$. The degree of these characters $\langle\lambda\rangle$ and $\langle\lambda\rangle'$ is [1,6]

$$2^{\left[\frac{(n-m)}{2}\right]}\frac{n!}{\prod_{i=1}^{m}\lambda!}\prod_{1\leq i<j\leq m}\frac{\lambda_i-\lambda_j}{\lambda_i+\lambda_j}$$

The decomposition matrix gives the relationships between the irreducible spin characters and projective indecomposable spin characters of S_n.

In this paper we determined the irreducible modular spin characters of the symmetric group S_{21} modulo 13 by using the method (r,r') -inducing (restricting) [3] to distribute the spin characters into p-blocks and use Morris-Humphreys theorem [4]. The Brauer trees for spin characters of S_n , $13 \le n \le 20$ modulo p=13 are found by Taban and Jawad [5].

2. Preliminaries

The fundamental theorem of the modular spin characters of symmetric groups S_n which distribute the spin irreducible characters into p-block is called Morris-Humphreys Theorem [4] Morris formulated this conjecture on how the irreducible spin characters of $\hat{S}_n$ are assigned into p-blocks [6] and proved by [7]. To formulate this theorem, the following definitions are given:

Definition: (2.1) Let $\lambda \mapsto n$ then the shifted young diagram $S(\lambda)$ of λ is the diagram obtained from the young diagram of λ by moving the i th row of λ $(i-1)$ position to the right.

There is a young diagram $\tilde{\lambda}$ with 2n nodes, called the shifted-symmetric diagram where

$\tilde{\lambda}$ = $(\lambda_1, \lambda_2, \ldots\ldots, \lambda_m \ ; \ \lambda_1 - 1, \lambda_2 - 1, \ldots\ldots, \lambda_m - 1)$

Example: if λ = (751)

```
          X X X X X X X
S(λ) =      X X X X X
              X
```

```
        0 X X X X X X X
        0 0 X X XX X
λ̃ =     0 0 0 X
        0 0
        0 0
        0 0
        0
```

Definition: (2.2) The p-bar of $S(\lambda)$ means bars of length P that is either

(1) Two rows which contain p-nodes or
(2) One row which contains the last p-nodes such that the resulting diagram gives a bar partition.

Definition: (2.3) A partition is a p-bar core or $\bar{\text{p}}$-core if no p-bar can be removed from it.

Theorem: (**Morris-Humphreys Theorem**) [4]

Let λ and μ be a bar partition such that $\lambda \neq \mu$ then $\langle \lambda \rangle$ and $\langle \mu \rangle$ are in the same p-block if and only if $\lambda(\bar{p}) = \mu(\bar{p})$ (where p is an odd prime). The associative irreducible spin characters $\langle \lambda \rangle$ and $\langle \lambda \rangle'$

are in the same p-block if $\lambda(\overline{p}) \neq \lambda$.

Example: Let $\lambda = \langle 621 \rangle, P = 5$

$(\tilde{\lambda})$ =

```
0 1 2 3 4 0 1
4 0 1 2
3 4 0 1
2
1
0
```

$\langle 621 \rangle \uparrow^{(2,4)} = \langle 721 \rangle + \langle 721 \rangle'$

$\langle 621 \rangle \uparrow^{(3,3)} = \langle 631 \rangle + \langle 631 \rangle'$

Definition: (2.4) Let λ be a bar partition of n then the p-residue of the (i, j)-node of $\tilde{\lambda}$ is the least non negative integer r such that $j - i \equiv r \pmod{p}$, $0 \le r \le p-1$

Let $\overline{r} = p - r + 1$ then $r + \overline{r} = 1 \pmod{p}$, $\overline{r}$ is called the conjugate residue to r .

We now denote by $\langle \lambda \rangle \uparrow^{(r,\overline{r})}$ all the diagrams obtained by adding one node with p-residue r or $\overline{r}$ to the $S(\lambda)$ component of the diagram $\langle \hat{\lambda} \rangle$. Then for $r = 0,1,2,\ldots,\dfrac{p+1}{2}$.

$\langle \hat{\lambda} \rangle \uparrow^{(r,\overline{r})}$ are called the diagrams $(r,\overline{r})-induced$ from $\langle \lambda \rangle$ conversely the process of deleting nodes with p-residue r or $\overline{r}$ is called $(r,\overline{r})-restriction$

Example: Let $\langle \lambda \rangle = \langle 531 \rangle, p = 3$

$(\tilde{\lambda})$ =

0	1	2	0	1	2	0
2	0	1	2	0	1	
1	2	0	1	2		
0	1	2				
2	0					
1						

$\langle \lambda \rangle \uparrow^{(0,1)} = \langle 631 \rangle + \langle 631 \rangle' + \langle 541 \rangle + \langle 541 \rangle'$

$\langle \lambda \rangle \uparrow^{(2,2)} = \langle 532 \rangle + \langle 532 \rangle'$

Now, if $\varphi = \sum d\lambda \langle \lambda \rangle + d\lambda' \langle \lambda' \rangle$ is projective indecomposable spin character of S_n (where $d\lambda' = 0$ if $\langle \lambda \rangle = \langle \lambda' \rangle$ then $\varphi \uparrow S_{n+1}$ is a projective spin character of S_{n+1} which is in general not indecomposable [3].

The following results are very useful to find the modular characters:

1. Every spin (modular, projective) character of S_n can be written as a linear combination with non-negative integer coefficients of the irreducible spin (irreducible modular, projective indecomposable) characters respectively [8]
2. Let H be a subgroup of the group G [9] then :
   (a) If φ is a modular (principle) character of a subgroup H of G, then $\varphi \uparrow G$ is a modular (principal) character of G (where $\uparrow$ denotes inducing)
   (b) If ψ is a modular (principal) character of group G, then $\psi \downarrow H$ is a modular (principal) character of a subgroup H. (where $\downarrow$ denotes the restricting)
3. Let G be a group of order $m = m_0 p^a$, where $(p, m_0) = 0$, if c is principal character of H then degree $c \equiv 0 \bmod p^a$ [10].
4. If c is a principal character of G for an odd prime p and all entries in c are divisible by positive integer q, then c/q is a principal character of G [9]
5. Let $(\alpha) = (\alpha_1, \alpha_2, \ldots\ldots, \alpha_m)$ be a bar partition of n not a p-bar core , let B be the block containing $\langle \alpha \rangle$ then
   (a) If $n - m - m_0$ is even then all irreducible modular spin characters in B are double
   (b) If $n - m - m_0$ is odd then all irreducible modular spin characters in B are associate (where m_0 the number of parts of α divisible by p). [11]

Notation

i.m.s. : irreducible modular spin characters

m.s. : modular spin characters

p.i.s . : Principal indecomposable spin character

p.s. : principal spin character

The Brauer tree of the symmetric group $\overline{S}_n$, p=13. The group S_{21} has 114 irreducible spin characters and S_{21} has 105 of $(13, \alpha)$ – regular classes then the decomposition matrix of the spin character of S_{21}, p=13 has 114 rows and 105 columns. [4]
There are fifty seven 13-block, (Morris and Humphreys theorem) $B_1, B_2, B_3, B_4, B_5, B_6$, 6 of their blocks of defect 1.
All the 51 remaining characters form their own blocks $B_7, B_8, B_9, \ldots.. B_{57}$ of defect 0 [9] which are irreducible modular spin characters.
The principal block B_1 (the block which contains the spin character $<n>$ or $<n>'$ where B_1 contains the irreducible spin characters
$\{<21>^*, <13,8>, <13,8>', <12,8,1>^*, <11,8,2>^*, <9,8,4>^*, <8,7,6>^*, <10,8,3>^*\}$ has 13 – bar core $<8>$

B_2 Contains the irreducible spin characters

$$\left\{\begin{array}{l}<20,1>\ ,<20,1>',<14,7>\ ,<14,7>',<13,7,1>^{*},\ <11,7,2,1>,\ <11,7,2,1>', \\ <10,7,3,1>,<10,7,3,1>',\ <9,7,4,1>,<9,7,4,1>',\ <8,7,5,1>,<8,7,5,1>'\end{array}\right\}$$

Has 13-bar core $<7,1>$

B_3 Contains the irreducible spin characters

$$\left\{\begin{array}{l}<19,2>\ ,<19,2>',<15,6>\ ,<15,6>',<13,6,2>^{*},\ <12,6,2,1>,\ <12,6,2,1>', \\ <10,6,3,2>,<10,6,3,2>',\ <9,6,4,2>,<9,6,4,2>',\ <8,6,5,2>,<8,6,5,2>'\end{array}\right\}$$

Has 13-bar core $<6,2>$

B_4 Contains the irreducible spin characters

$$\left\{\begin{array}{l}<18,3>\ ,<18,3>',<16,5>\ ,<16,5>',<13,5,3>^{*},\ <12,5,3,1>,\ <12,5,3,1>',<11,5,3,2>, \\ <11,5,3,2>',\ <9,5,4,3>,<9,5,4,3>',\ <7,6,5,3>,<7,6,5,3>'\end{array}\right\}$$

Has 13-bar core $<5,3>$

B_5 Contains the irreducible spin characters

$$\left\{<18,2,1>^{*},<15,5,1>^{*},<14,5,2>^{*},<13,5,2,1>,<13,5,2,1>',<10,5,3,2,1>^{*},<9,5,4,2,1>^{*},<7,6,5,2,1>^{*},\right\}$$

Has 13-bar core $<5,2,1>$

B_6 Contains the irreducible spin characters

$$\left\{<17,3,1>^{*},<16,4,1>^{*},<14,4,3>^{*},<13,4,3,1>,<13,4,3,1>',<11,4,3,2,1>^{*},<8,5,4,3,1>^{*},<7,6,4,3,1>^{*}\right\}$$

Has 13-bar core $<4,3,1>$

Lemma (3.1)

The Brauer tree for Principal block B_1 is:

$$<21>^{*} - <13,8> = <13,8>' - <12,8,1>^{*} - <11,8,2>^{*} - <10,8,3>^{*} - <9,8,4>^{*} - <8,7,6>^{*}$$

Proof :

$\deg\{ <21>^*,<12,8,1>^*,<10,8,3>^*,<8,7,6>^* \} \equiv 10 \mod 13$;

$\deg\{ (<13,8>+<13,8>'),<11,8,2>^*,<9,8,4>^* \} \equiv -10 \mod 13$.

By using $(r,\overline{r})-inducing$ for p.i.s of S_{20} see (appendix I) to S_{21} we have

$J_{31} \uparrow^{(8,6)} S_{21} = <20> + <13,7>^{*} \uparrow^{(8,6)} S_{21} = <21>^{*}+<13,8>+<13,8>' = d_1$

$J_{33} \uparrow^{(8,6)} S_{21} = <13,7>^{*}+<12,7,1> \uparrow^{(8,6)} S_{21} = <13,8>+<13,8>'+<12,8,1>^{*} = d_2$

$J_{35} \uparrow^{(8,6)} S_{21} = <12,7,1>+<11,7,2> \uparrow^{(8,6)} S_{21} = <12,8,1>^{*}+<11,8,2>^{*} = d_3$

$J_{37} \uparrow^{(8,6)} S_{21} = <11,7,2>+<10,7,3> \uparrow^{(8,6)} S_{21} = <11,8,2>^{*}+<10,8,3>^{*} = d_4$

$J_{39} \uparrow^{(8,6)} S_{21} = <10,7,3> +<9,7,4> \uparrow^{(8,6)} S_{21} = <10,8,3>^{*}+<9,8,4>^{*} = d_5$

$J_{41} \uparrow^{(8,6)} S_{21} = <9,7,4> +<8,7,5> \uparrow^{(8,6)} S_{21} = <9,8,4>^{*}+<8,7,6>^{*} = d_6$

So we have the Brauer tree for B_1 and the decomposition matrix of this block $D_{21,13}{}^{(1)}$ in Table (1) "

Lemma (3.2)

The Brauer tree for the block B_2 is:

```
<20,1> − <14,7>                    <11,7,2,1> − <10,7,3,1> − <9,7,4,1> − <8,7,5,1>
                   \             /
                     <13,7,1>*
                   /             \
<20,1>′ − <14,7>′                  <11,7,2,1>′ − <10,7,3,1>′ − <9,7,4,1>′ − <8,7,5,1>′
```

Proof :

$\deg\{<14,7>,<14,7>',<11,7,2,1>,<11,7,2,1>',<9,7,4,1>,<9,7,4,1>'\} \equiv -4 \mod 13$

$\deg\{<20,1>,<20,1>',<13,7,1>^{*},<10,7,3,1>,<10,7,3,1>',<8,7,5,1>,<8,7,5,1>'\} \equiv 4 \mod 13$

By using $(r,\bar{r})-inducing$ for p.i.s of S_{20} see (appendix I) to S_{21} we have

$J_{13}\uparrow^{(7,7)} S_{21} = <19,1>^{*}+<14,6>^{*}\uparrow^{(7,7)} S_{21} = <20,1>+<20,1>'<14,7>+<14,7>' = k_1$

$J_{31}\uparrow^{(1,0)} S_{21} = <20>+<13,7>^{*}\uparrow^{(1,0)} S_{21} = <20,1>+<14,7>+<14,7>'+<13,7,1>^{*} = k_2$

$J_{32}\uparrow^{(1,0)} S_{21} = <20>'+<13,7>^{*}\uparrow^{(1,0)} S_{21} = <20,1>'+<14,7>+<14,7>'+<13,7,1>^{*} = k_3$

$J_{35}\uparrow^{(1,0)} S_{21} = <12,7,1>+<11,7,2>\uparrow^{(1,0)} S_{21} = <13,7,1>^{*}+<11,7,2,1> = d_{11}$

$J_{36}\uparrow^{(1,0)} S_{21} = <12,7,1>'+<11,7,2>'\uparrow^{(1,0)} S_{21} = <13,7,1>^{*}+<11,7,2,1>' = d_{12}$

$J_{37}\uparrow^{(1,0)} S_{21} = <11,7,2>+<10,7,3>\uparrow^{(1,0)} S_{21} = <11,7,2,1>+<10,7,3,1> = d_{13}$

$J_{38}\uparrow^{(1,0)} S_{21} = <11,7,2>'+<10,7,3>'\uparrow^{(1,0)} S_{21} = <11,7,2,1>'+<10,7,3,1>' = d_{14}$

$J_{39}\uparrow^{(1,0)} S_{21} = <10,7,3>+<9,7,4>\uparrow^{(1,0)} S_{21} = <10,7,3,1>+<9,7,4,1> = d_{15}$

$J_{40}\uparrow^{(1,0)} S_{21} = <10,7,3>'+<9,7,4>'\uparrow^{(1,0)} S_{21} = <10,7,3,1>'+<9,7,4,1>' = d_{16}$

$J_{41}\uparrow^{(1,0)} S_{21} = <9,7,4>+<8,7,5>\uparrow^{(1,0)} S_{21} = <9,7,4,1>+<8,7,5,1> = d_{17}$

$J_{42}\uparrow^{(1,0)} S_{21} = <9,7,4>'+<8,7,5>'\uparrow^{(1,0)} S_{21} = <9,7,4,1>'+<8,7,5,1>' = d_{18}$

$<14,7,1>\ \downarrow_{S_{21}}^{(1,0)} = <13,7,1>^{*}+<14,7> = d_9$ Since $<14,7,1>$ i.m in S_{22}

$<14,7,1>'\ \downarrow_{S_{21}}^{(1,0)} = <13,7,1>^{*}+<14,7>' = d_{10}$ Since $<14,7,1>'$ i.m in S_{22}

But $k_1 = k_2 + k_3 - d_9 - d_{10}$ then $k_2 - d_{10} = d_7$, $k_3 - d_9 = d_8$

So we have the Brauer tree for B_2 [12] and the decomposition matrix for this block $D_{21,13}^{(2)}$ in Table (2)

Lemma (3.3)

The Brauer tree for the block B_3 is:

```
<19,2> – <15,6>                     <12,6,2,1> – <10,6,3,2> – <9,6,4,2> – <8,6,5,2>
                  \               /
                    <13,6,2>*
                  /               \
<19,2>′ – <15,6>′                   <12,6,2,1>′ – <10,6,3,2>′ – <9,6,4,2>′ – <8,6,5,2>′
```

Proof :

$\deg\{<19,2>,<19,2>', <13,6,2>^*,<10,6,3,2>,<10,6,3,2>',<8,6,5,2>,<8,6,5,2>'\} \equiv -8 \mod 13$

$\deg \{<15,6>,<15,6>',<12,6,2,1>,<12,6,2,1>',<9,6,4,2>,<9,6,4,2>'\} \equiv 8 \mod 13$

Now, by using $(r,\bar{r})-inducing$ of p.i.s of S_{20} to S_{21} see (appendix I) $D_{20,13}$ we have

$J_{13}\uparrow^{(2,12)} S_{21} = <19,1>^* + <14,6>^* \uparrow^{(2,12)} S_{21} = <19,2>+<19,2>'+<15,6>+<15,6>' = k_1 = d_{19}+d_{20}$

$J_{14}\uparrow^{(2,12)} S_{21} = <14,6>^* + <13,6,1>+<13,6,1>' \uparrow^{(2,12)} S_{21} = <15,6>+<15,6>'+2<13,6,2>^* = k_2 = d_{21}+d_{22}$

$J_{15}\uparrow^{(2,12)} S_{21} = <13,6,1>+<13,6,1>'+<11,6,2,1>^* \uparrow^{(2,12)} S_{21} = 2<13,6,2>^*+<12,6,2,1>+<12,6,2,1>' = k_3 = d_{23}+d_{24}$

$J_{16}\uparrow^{(2,12)} S_{21} = <11,6,2,1>^* + <10,6,3,1>^* \uparrow^{(2,12)} S_{21} = <12,6,2,1>+<12,6,2,1>'+<10,6,3,2>+<10,6,3,2>' = k_4 = d_{25}+d_{26}$

$J_{17}\uparrow^{(2,12)} S_{21} = <10,6,3,1>^* + <9,6,4,1>^* \uparrow^{(2,12)} S_{21} = <10,6,3,2>+<10,6,3,2>'+<9,6,4,2>+<9,6,4,2>' = k_5 = d_{27}+d_{28}$

$J_{18}\uparrow^{(2,12)} S_{21} = <9,6,4,1>^* + <8,6,5,1>^* \uparrow^{(2,12)} S_{21} = <9,6,4,2>+<9,6,4,2>'+<8,6,5,2>+<8,6,5,2>' = k_6 = d_{29}+d_{30}$

Since $<19,2> \neq <19,2>'$, $<15,6> \neq <15,6>'$, $<12,6,2,1> \neq <12,6,2,1>'$, $<10,6,3,2> \neq <10,6,3,2>'$, $<9,6,4,2> \neq <9,6,4,2>'$ and $<8,6,5,2> \neq <8,6,5,2>'$

On $(13,\alpha)$ regular classes then k_1, k_2, k_3, k_4, k_5, and k_6 are splits (respectively) so we have the Brauer tree for B_3 and the decomposition matrix for this block $D_{21,13}^{(3)}$ in Table (3)

Lemma (3.4)

The Brauer tree for the block B_4 is:

```
<18,3> – <16,5>                     <12,5,3,1> – <11,5,3,2> – <9,5,4,3> – <7,6,5,3>
                  \               /
                    <13,5,3>*
                  /               \
<18,3>′ – <16,5>′                   <12,5,3,1>′ – <11,5,3,2>′ – <9,5,4,3>′ – <7,6,5,3>′
```

Proof :

$$\deg\{<18,3>\ ,\ <18,3>',<13,5,3>^{*},<11,5,3,2>\ ,\ <11,5,3,2>',<7,6,5,3>\ ,\ <7,6,5,3>'\} \equiv -8 \mod 13$$

$$\deg\ \{<16,5>\ ,\ <16,5>',<12,5,3,1>\ ,\ <12,5,3,1>',<9,5,4,3>\ ,\ <9,5,4,3>'\} \equiv 8 \mod 13$$

Now, by $(r,\bar{r})-inducing$ of p.i.s of S_{20} to S_{21} see (appendix I) $D_{20,3}$ we have

$J_{19}\uparrow^{(3,11)} S_{21} = <18,2>^{*} + <15,5>^{*}\uparrow^{(3,11)} S_{21} = <18,3> + <18,3>' + <16,5> + <16,5>' = k_1 = d_{31}+d_{32}$

$J_{20}\uparrow^{(3,11)} S_{21} = <15,5>^{*} + <13,5,2> + <13,5,2>' + \uparrow^{(3,11)} S_{21} = <16,5> + <16,5>' + 2<13,5,3>^{*} = k_2 = d_{33}+d_{34}$

$J_{21}\uparrow^{(3,11)} S_{21} = <13,5,2> + <13,5,2>' + <12,5,2,1>^{*}\uparrow^{(3,11)} S_{21} = 2<13,5,3>^{*} + <12,5,3,1> + <12,5,3,1>' = k_3 = d_{35}+d_{36}$

$J_{22}\uparrow^{(3,11)} S_{21} = <12,5,2,1>^{*} + <10,5,3,2>\uparrow^{(3,11)} S_{21} = <12,5,3,1> + <12,5,3,1>' + <11,5,3,2> + <11,5,3,2>' = k_4 = d_{37}+d_{38}$

$J_{23}\uparrow^{(3,11)} S_{21} = <10,5,3,2>^{*} + <9,5,4,2>^{*}\uparrow^{(3,11)} S_{21} = <11,5,3,2> + <11,5,3,2>' + <9,5,4,3> + <9,5,4,3>' = k_5 = d_{39}+d_{40}$

$J_{24}\uparrow^{(3,11)} S_{21} = <9,5,4,2>^{*} + <7,6,5,2>^{*}\uparrow^{(3,11)} S_{21} = <9,5,4,3> + <9,5,4,3>' + <7,6,5,3> + <7,6,5,3>' = k_6 = d_{41}+d_{42}$

Since $<18,3> \neq <18,3>'$, $<16,5> \neq <16,5>'$, $<12,5,3,1> \neq <12,5,3,1>'$, $<11,5,3,2> \neq <11,5,3,2>'$, $<9,5,4,3> \neq <9,5,4,3>'$ and $<7,6,5,3> \neq <7,6,5,3>'$

On $(13,\alpha)$ regular classes then k_1, k_2, k_3, k_4, k_5, and k_6 are splits (respectively) then we have the Brauer tree for B_4 and the decomposition matrix for this block $D_{21,13}^{(4)}$ in Table (4)

Lemma (3.5)

The Brauer tree for the block B_5 is:

$<18,2,1>^{*} - <15,5,1>^{*} - <14,5,2>^{*} - <13,5,2,1> = <13,5,2,1>' - <10,5,3,2,1>^{*} - <9,5,4,2,1>^{*} - <7,6,5,2,1>^{*}$

Proof:

$$\deg\{<18,2,1>^{*},<14,5,2>^{*},<10,5,3,2,1>^{*},<7,6,5,2,1>^{*}\} \equiv 2 \mod 13$$

$$\deg\ \{<15,5,1>^{*},<13,5,2,1> = <13,5,2,1>',<9,5,4,2,1>^{*}\} \equiv -2 \mod 13$$

Now, by using $(r,\bar{r})-inducing$ of p.i.s of S_{20} to S_{21} see table $D_{20,13}$ we have

$J_{19}\uparrow^{(1,0)} S_{21} = <18,2>^{*} + <15,5>^{*}\uparrow^{(1,0)} S_{21} = <18,2,1>^{*} + <15,5,1>^{*} = d_{43}$

$J_{20}\uparrow^{(1,0)} S_{21} = <15,5>^{*} + <13,5,2> + <13,5,2>'\uparrow^{(1,0)} S_{21} = <15,5,1>^{*} + 2<14,5,2>^{*} + <13,5,2,1> + <13,5,2,1>' = k_1$

$J_{21}\uparrow^{(1,0)} S_{21} = <13,5,2> + <13,5,2>' + <12,5,2,1>^{*}\uparrow^{(1,0)} S_{21} = 2<14,5,2>^{*} + 2<13,5,2,1> + 2<13,5,2,1>' = 2k_2 = 2d_{45}$

$J_{22}\uparrow^{(1,0)} S_{21} = <12,5,2,1>^{*} + <10,5,3,2>^{*}\uparrow^{(1,0)} S_{21} = <13,5,2,1> + <13,5,2,1>' + <10,5,3,2,1>^{*} = d_{46}$

$J_{23}\uparrow^{(1,0)} S_{21} = <10,5,3,2>^{*} + <9,5,4,2>^{*}\uparrow^{(1,0)} S_{21} = <10,5,3,2,1>^{*} + <9,5,4,2,1>^{*} = d_{47}$

$J_{24}\uparrow^{(1,0)} S_{21} = <9,5,4,2>^{*} + <7,6,5,2>^{*}\uparrow^{(1,0)} S_{21} = <9,5,4,2,1>^{*} + <7,6,5,2,1>^{*} = d_{48}$

Since $<15,5,2>^* \downarrow^{(2,12)} S_{21} = <15,5,1>^* + <14,5,2>^*$ thus $k_1 - k_2 = <15,5,1>^* + <14,5,2>^* = d_{44}$ i.m. in S_{21} and $k_2 = d_{45}$ so we have the Brauer tree for B_5 and the decomposition matrix for this block $D_{21,13}^{(5)}$ in Table (5)

Lemma (3.6)

The Brauer tree for the block B_6 is:

$$<17,3,1>^* - <16,4,1>^* - <14,4,3>^* - <13,4,3,1> = <13,4,3,1>' - <11,4,3,2,1>^* - <8,5,4,3,1>^* - <7,6,4,3,1>^*$$

Proof :

$$\deg\{<17,3,1>^*, <14,4,3>^*, <11,4,3,2,1>^*, <7,6,4,3,1>^*\} \equiv 8 \mod 13$$

$$\deg\{<16,4,1>^*, <13,4,3,1> = <13,4,3,1>', <8,5,4,3,1>^*\} \equiv -8 \mod 13$$

Now, by using $(r,\bar{r})-inducing$ of p.i.s of S_{20} to S_{21} see (appendix I) $D_{20,3}$ we have

$J_{25} \uparrow^{(1,0)} S_{21} = <17,3>^* + <16,4>^* \uparrow^{(1,0)} S_{21} = <17,3,1>^* + <16,4,1>^* = d_{49}$

$J_{27} \uparrow^{(1,0)} S_{21} = <13,4,3> + <13,4,3>' + <12,4,3,1>^* \uparrow^{(1,0)} S_{21} = 2<14,4,3>^* + 2<13,4,3,1> + 2<13,4,3,1>' = 2d_{51}$

$J_{28} \uparrow^{(1,0)} S_{21} = <12,4,3,1>^* + <11,4,3,2>^* \uparrow^{(1,0)} S_{21} = <13,4,3,1> + <13,4,3,1>' + <11,4,3,2,1>^* = d_{52}$

$J_{29} \uparrow^{(1,0)} S_{21} = <11,4,3,2>^* + <8,5,4,3>^* \uparrow^{(1,0)} S_{21} = <11,4,3,2,1>^* + <8,5,4,3,1>^* = d_{53}$

$J_{30} \uparrow^{(1,0)} S_{21} = <8,5,4,3>^* + <7,6,4,3>^* \uparrow^{(1,0)} S_{21} = <8,5,4,3,1>^* + <7,6,4,3,1>^* = d_{54}$

Moreover, in [5] $J_3 = <15,4,1> + <14,4,2>$ p.i.s of S_{20} then

$J_3 \uparrow^{(3,11)} S_{21} = <15,4,1> + <14,4,2> \uparrow^{(3,11)} S_{21} = <16,4,1>^* + <14,4,3>^* = d_{50}$

So we have the Brauer tree for B_6 and the decomposition matrix for this block $D_{21,13}^{(6)}$ in Table (6)

Appendix I

The decomposition matrix $D_{20,13}$ for the spin characters of S_{20} , p=13

The spin characters	The decomposition matrix for the block B_5											
<20>	1											
<20>′		1										
<13,7>*	1	1	1	1								
<12,7,1>			1		1							
<12,7,1>′				1		1						
<11,7,2>					1		1					
<11,7,2>′						1		1				
<10,7,3>							1		1			
<10,7,3>′								1		1		
<9,7,4>									1		1	
<9,7,4>′										1		1
<8,7,5>											1	
<8,7,5>′												1
	J_{31}	J_{32}	J_{33}	J_{34}	J_{35}	J_{36}	J_{37}	J_{38}	J_{39}	J_{40}	J_{41}	J_{42}

The spin characters	The decomposition matrix for the block B_2					
<19,1>*	1					
<14,6>*	1	1				
<13,6,1>		1	1			
<13,6,1>′		1	1			
<11,6,2,1>*			1	1		
<10,6,3,1>*				1	1	
<9,6,4,1>*					1	1
<8,6,5,1>*						1
	J_{13}	J_{14}	J_{15}	J_{16}	J_{17}	J_{18}

The spin characters	The decomposition matrix for the block B_3					
<18,2>*	1					
<15,5>*	1	1				
<13,5,2>		1	1			
<13,5,2>′		1	1			
<12,5,2,1>*			1	1		
<10,5,3,2>*				1	1	
<9,5,4,2>*					1	1
<7,6,5,2>*						1
	J_{19}	J_{20}	J_{21}	J_{22}	J_{23}	J_{24}

The spin characters	The decomposition matrix for the block B_4					
<17,3>*	1					
<16,4>*	1	1				
<13,4,3>		1	1			
<13,4,3>′		1	1			
<12,4,3,1>*			1	1		
<11,4,3,2>*				1	1	
<8,5,4,3>*					1	1
<7,6,4,3>*						1
	J_{25}	J_{26}	J_{27}	J_{28}	J_{29}	J_{30}

Table (1), $D_{21,13}^{(1)}$

The spin characters	The decomposition matrix for the block B_1					
<21>*	1					
<13,8>	1	1				
<13,8>′	1	1				
<12,8,1>*		1	1			
<11,8,2>*			1	1		
<10,8,3>*				1	1	
<9,8,4>*					1	1
<8,7,6>*						1
	d_1	d_2	d_3	d_4	d_5	d_6

Table (2), $D_{21,13}^{(2)}$

The spin characters	The decomposition matrix for the block B_2											
<20,1>	1											
<20,1>′		1										
<14,7>	1		1									
<14,7>′		1		1								
<13,7,1>*			1	1	1	1						
<11,7,2,1>					1		1					
<11,7,2,1>′						1		1				
<10,7,3,1>							1		1			
<10,7,3,1>′								1		1		
<9,7,4,1>									1		1	
<9,7,4,1>′										1		1
<8,7,5,1>											1	
<8,7,5,1>′												1
	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}

Table (3), $D_{21,13}^{(3)}$

The spin characters	The decomposition matrix for the block B_3											
<19,2>	1											
<19,2>′		1										
<15,6>	1		1									
<15,6>′		1		1								
<13,6,2>*			1	1	1	1						
<12,6,2,1>					1		1					
<12,6,2,1>′						1		1				
<10,6,3,2>							1		1			
<10,6,3,2>′								1		1		
<9,6,4,2>									1		1	
<9,6,4,2>′										1		1
<8,6,5,2>											1	
<8,6,5,2>′												1
	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

Table (4), $D_{21,13}^{(4)}$

The spin characters	The decomposition matrix for the block B_4											
<18,3>	1											
<18,3>′		1										
<16,5>	1		1									
<16,5>′		1		1								
<13,5,3>*			1	1	1	1						
<12,5,3,1>					1		1					
<12,5,3,1>′						1		1				
<11,5,3,2>							1		1			
<11,5,3,2>′								1		1		
<9,5,4,3>									1		1	
<9,5,4,3>′										1		1
<7,6,5,3>											1	
<7,6,5,3>′												1
	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}	d_{41}	d_{42}

Table (5), $D_{21,13}^{(5)}$

The spin characters	The decomposition matrix for the block B_5					
<18,2,1>*	1					
<15,5,1>*	1	1				
<14,5,2>*		1	1			
<13,5,2,1>			1	1		
<13,5,2,1>′			1	1		
<10,5,3,2,1>*				1	1	
<9,5,4,2,1>*					1	1
<7,6,5,2,1>*						1
	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}

Table (6), $D_{21,13}^{(6)}$

The spin characters	The decomposition matrix for the block B_6					
<17,3,1>*	1					
<16,4,1>*	1	1				
<14,4,3>*		1	1			
<13,4,3,1>			1	1		
<13,4,3,1>′			1	1		
<11,4,3,2,1>*				1	1	
<8,5,4,3,1>*					1	1
<7,6,4,3,1>*						1
	d_{49}	d_{50}	d_{51}	d_{52}	d_{53}	d_{54}

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شجرات براور للزمره التناظريه S_{21} معيار p=13

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المستخلص

الهدف الرئيسي في هذا البحث هو ايجاد شجرات براور للزمره التناظريه S_{21} معيار $p = 13$ والتي تعطي المشخصات الاسقاطيه المعياريه ل S_{21} معيار $p = 13$