Fixed points theorems for ciric' mappings in partial b-metric space

Amal M. Hashim and Haneen A. Bakry
Department of Mathematics, College of Sciences, University of Basrah,Basrah-Iraq

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Abstract

The main purpose of this paper, is to introduce and study the common fixed point by using the concept of partial metric space and combine with class of b-metric space under a contractive condition which introduce LJ-B. Ciric. Our results improve and unify a multitude of fixed point theorems and generalize some recent results in partial b-metric spaces.

Keywords : common fixed point, weak* compatible maps, Partial b-metric space

1. Introduction

In Czerwik [1] introduced the concept of b-metric space as a generalization of metric space and proved the Banach Contraction principle in b-metric space. In Matthew [2] introduced the notion of partial metric space as a generalization of metric space in which each object does not necessarily have a zero distance from itself.

Recently in Shukla [3] introduced the notion of partial b-metric space as a generalization of partial metric space and b-metric space, and he proved the fixed point theorem of Banach contraction principle and Kannan type mapping in partial b-metric space.

In this paper, we prove some fixed point in partial b-metric space for generalized contraction which introduced by Ciric [4] (see for instance ([5]-[12]) and refernce thereof)

2. Preliminaries

we recall some definitions and notions of partial b-metric space.

Definition 2.1 *[1] A b-metric on a nonempty set X is a self map $d: X^2 \to R^+$ satisfying the following conditions:*

(bM1) $d(x, y) = 0$ if and only if $x = y$, for every x, y in X;

(bM2) $d(x, y) = d(y, x)$,

(bM3) There exist areal number $b \geq 1$ such that $d(x, y) \leq b[d(x, z) + d(z, y)]$, for every x, y, z in X;

the pair (X, d) is called a b-metric space (b.M.S) a generalization of usual metric space.

Definition 2.2 *[6] A partial metric on a nonempty set X, is a self map* $p: X^2 \to R^+$ satisfying the following axioms:

(pM1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,(separation axiom)

(pM2) $0 \leq p(x, x) \leq p(x, y)$, (non-negativity and small self-distance)

(pM3) $p(x, y) = p(y, x)$, (symmetry)

(pM4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$, (triangular inequality) for all x, y, z in X. then (X, p) is called a partial metric space for short $(P.M.S)$ It is clearly that, every metric is a partial metric.

Definition 2.3 *[7] Let X be a nonempty set, $b \geq 1$ be a given real number and let $\rho: X^2 \to R^+$ be a self map such that for every x, y, z in X, the following conditions hold:*
(pbM1) $x = y$ if and only if $\rho(x, x) = \rho(x, y) = \rho(y, y)$,
(pbM2) $\rho(x, x) \leq \rho(x, y)$,
(pbM3) $\rho(x, y) = \rho(y, x)$,

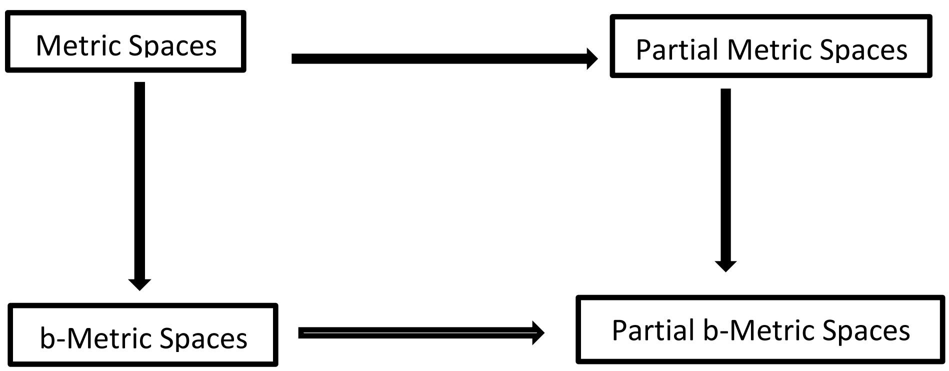
$$\rho(x, y) \leq b[\rho(x, z) + \rho(z, y)] - \rho(z, z)$$

Then the pair (X, ρ) is called Partial b-Metric Space (Pb.M.S) for short.

Remark:*[7]*

In Partial b-metric Space (X, ρ), if $x, y, \in X$ and $\rho(x, y) = 0$, then $x = y$ but the converse may not be true.

we remark that every Partial b-Metric defines a b-Metric d, where $d(x, y) = 2\rho(x, y) - \rho(x, x) - \rho(y, y)$ for all $x, y, z \in X$.



Now we define the convergence of a sequence and Cauchy sequence in partial b-metric space.

Definition 2.4 *[7] Let* (X, ρ)*be* Partial b-metric spaces, *Let* $\{x_n\}$ *be any sequence in X, and $x \in X$. Then:*

1. The $\{x_n\}$ sequence is said to be convergent and convergent to x, if
$$\lim_{n\to\infty} \rho(x_n, x) = \rho(x, x)$$
2. The $\{x_n\}$ sequence is said to be Cauchy sequence in (X, ρ), if
$$\lim_{n,m\to\infty} \rho(x_n, x_m)$$
exists and is finite;
3. (X, ρ) is said to be a complete partial b-metric space if for every Cauchy sequence $\{x_n\}$ in X, there exists $x \in X$ such that
$$\lim_{n,m\to\infty} \rho(x_n, x_m) = \lim_{n\to\infty} \rho(x_n, x) = \rho(x, x),$$

Note that in a partial b-metric space the limit of convergent sequence may not be unique[7].

Example 2.5 *Let $X = R^+$, and $\rho: X^2 \to R^+$ be a self map defined by*

$$\rho(x,y) = [\max\{x,y\}]^4 + |x-y|^4$$

for all $x, y \in X$.

Then (X,ρ) is partial b-metric space with $b = 2^4 > 1$. but it is neither a b-metric nor partial metric space. Indeed, for any $x > 0$ we have $\rho(x,x) = x^p \neq 0$; therefore, ρ is not a b-metric on X. Also for $x = 6,\ y = 2,\ z = 3$ we have $\rho(x,y) = 6^4 + 4^4$ and $\rho(x,z) + \rho(z,y) - \rho(z,z) = 6^4 + 3^4 + 3^4 + 1^4 - 3^4 = 6^4 + 3^4 + 1^4$ so

$\rho(x,y) > \rho(x,z) + \rho(z,y) - \rho(z,z)$ for all $b = 2^4$;therefore, ρ is not a partial metric on X.

Definition 2.6 *[8] Two self map f and g of a non empty set X are called weakly compatible if they commute at coincidence points i.e,*

$fgz = gfz$ for every $z \in X$ whenever, $fz = gz$

Definition 2.7 *[8] Two self map f and g of a non empty set X are called weakly* compatible if they commute at one of their coincidence points that is, if there exists a point $x \in X$ such that $fx = Tx$ then $fTx = Tfx$*

at weakly* compatible maps are more general than the weakly compatible maps for more details see,[8].

In (2014) Shukla [3] introduced the concept of partial b-metric space as a generalization of partial metric and b-metric spaces and proved Banach contraction principle in partial b-metric space.

3. Main Results

Theorem 3.1 *Let (X,ρ) be a complete* partial b-metric Space *with $b \geq 1$ and* $f: X \to X$ be a self map satisfying the following condition:

$$\rho(fx, fy)) \leq \lambda M(x,y), \tag{1}$$

where

$$M(x,y) = \max\{\rho(x,y), \rho(x,fx), \rho(y,fy), \tfrac{1}{2}[\rho(x,fy) + \rho(y,fx)]\}.$$

and $\lambda \in [0, \frac{1}{2b})$, $x, y \in X$.

Then, f posses a unique fixed point u and $\rho(u,u) = 0$.

Proof. First we have to show that if fixed point of f exists then it is unique. Let $u, v \in X$ be two distinct, fixed points of f, that is, $fu = u \neq fv = v$. It follow from (1) that

$$\rho(u,v) = \rho(fu,fv) \leq \lambda \max\{\rho(u,v), \rho(u,fu), \rho(v,fv), \tfrac{1}{2}[\rho(u,fv) + \rho(v,fu)]\}$$

$$= \lambda \max\{\rho(u,v), \rho(u,u), \rho(v,v), \tfrac{1}{2}[\rho(u,v) + \rho(v,u)]\}$$

$$= \lambda \max\{\rho(u,v), 0, 0, \rho(u,v)\}$$

$$\leq \tfrac{1}{2b}\rho(u,v) < \rho(u,v),$$

a contradiction. Thus we have $u = v$.

Next, we have to show the existence of fixed point. Let $x_0 \in X$ be arbitrary, set $x_{n+1} = fx_n$. if $x_n = x_{n+1}$ for some $n \in N$, then $x_n = fx_n$, x_n is a fixed point of f.

suppose, further, that $x_n \neq x_{n+1}$ for all $n \in N$, for the sake of convenience assume that $\rho_n = \rho(x_n, x_{n+1})$. We claim that $\rho_n < \rho_{n-1}$.

$$\rho_n = \rho(x_n, x_{n+1}) = \rho(fx_{n-1}, fx_n) \leq \lambda M(x_{n-1}, x_n)$$
$$\leq \lambda \max\{\rho(x_{n-1}, x_n), \rho(x_n, fx_n), \rho(x_{n-1}, fx_{n-1}),$$
$$\frac{1}{2}[\rho(x_{n-1}, fx_n) + \rho(x_n, fx_{n-1})]\}$$
$$= \lambda \max\{\rho(x_{n-1}, x_n), \rho(x_n, x_{n+1}), \rho(x_{n-1}, x_n),$$
$$\frac{1}{2}[\rho(x_{n-1}, x_{n+1}) + \rho(x_n, x_n)]\}$$

There is three cases

1. If
$$M(x_{n-1}, x_n) = \rho(x_n, x_{n+1})$$
then
$$\rho(x_n, x_{n+1}) \leq \lambda \rho(x_n, x_{n+1}) < \rho(x_n, x_{n+1})$$
which is contradiction.

2. If
$$M(x_{n-1}, x_n) = \rho(x_{n-1}, x_n),$$
then
$$\rho(x_n, x_{n+1}) \leq \lambda \rho(x_{n-1}, x_n) < \rho(x_{n-1}, x_n)$$

3. If
$$M(x_{n-1}, x_n) = \frac{1}{2}[\rho(x_{n-1}, x_{n+1}) + \rho(x_n, x_n)],$$
then
$$\rho(x_n, x_{n+1}) \leq \frac{\lambda}{2}[\rho(x_{n-1}, x_{n+1}) + \rho(x_n, x_n)].$$
from partial b-metric triangular property
$$\frac{\lambda}{2}[\rho(x_{n-1}, x_{n+1}) + \rho(x_n, x_n)] \leq \frac{\lambda}{2}\mathrm{b}[\rho(x_{n-1}, x_n)$$
$$+\rho(x_n, x_{n+1})] - \frac{\lambda}{2}\rho(x_n, x_n) + \frac{\lambda}{2}\rho(x_n, x_n)$$
$$= \frac{\lambda}{2}\mathrm{b}[\rho_{n-1} + \rho_n]$$

where $\lambda \in [0, \frac{1}{2b})$, let $\alpha = \frac{\lambda b}{2}$, then $\rho_n \leq \alpha[\rho_{n-1} + \rho_n]$, where $\alpha \in [0, \frac{1}{4})$.

Therefore, $\rho_n \leq \beta \rho_{n-1}$, where $\beta = \frac{\alpha}{1-\alpha}$.

Repeating this process, we have $\rho_n \leq \beta^n \rho_0$

$$\rho(x_{n+1}, x_n) \leq \beta^n \rho(x_1, x_0). \quad (2)$$

for all $n \geq 0$ There fore $\lim \rho_n = 0$.

Now we will show that $\{x_n\}$ is a cauchy sequence.It follow from (1) that for $n, m \in N n < m$

$$\rho(x_n, x_m) \leq b[\rho(x_n, x_{n+1}) + \rho(x_{n+1}, x_m)] - \rho(x_{n+1}, x_{n+1})$$
$$\leq b\rho(x_n, x_{n+1}) + b^2[\rho(x_{n+1}, x_{n+2}) + \rho(x_{n+2}, x_m)] - b\rho(x_{n+2}, x_{n+2})$$
$$\leq b\rho(x_n, x_{n+1}) + b^2\rho(x_{n+1}, x_{n+2}) + b^3\rho(x_{n+2}, x_{n+3}) + \ldots + b^{m-n}\rho(x_{m-1}, x_m)$$

by using (2) we obtain

$$\rho(x_n, x_m) \leq b\beta^n \rho(x_1, x_0) + b^2\beta^{n+1}\rho(x_1, x_0)$$
$$+b^3\beta^{n+2}\rho(x_1, x_0) + \ldots + b^{m-n}\beta^{m-1}\rho(x_1, x_0)$$
$$\leq b\beta^n[1 + b\beta + (b\beta)^2 + (b\beta)^3 + \ldots]\rho(x_1, x_0)$$

$$= \frac{b\beta^n}{1-b}\rho(x_1, x_0)$$

since $\alpha = \frac{\lambda b}{2}, \lambda \in [0, \frac{1}{2b})$ and $\beta = \frac{\alpha}{1-\alpha}$.Then we have

$$\lim_{n,m\to\infty} \rho(x_n, x_m) = 0$$

Therefor $\{x_n\}$ is a Cauchy sequence in X. Since X is a complete metric space there exists $z \in X$ such that

$$\lim_{n\to\infty} \rho(x_n, z) = \lim_{n,m\to\infty} \rho(x_n, x_m) = \rho(z, z) = 0 \quad (3)$$

we have to show z is a fixed point of f.

$$\rho(z, fz) \leq b[\rho(z, x_{n+1}) + \rho(x_{n+1}, fz)] - \rho(x_{n+1}, x_{n+1})$$
$$\leq b[\rho(z, x_{n+1}) + \rho(fx_n, fz)]$$
$$\leq b\rho(z, x_{n+1}) + 2\lambda \max\{\rho(x_n, z), \rho(x_n, fx_n), \rho(z, fz),$$
$$\frac{1}{2}[\rho(x_n, fz) + \rho(z, fx_n)]\}$$

$$\leq b[\rho(z, x_{n+1}) + \lambda b \max\{\rho(x_n, z), \rho(x_n, x_n), \rho(z, z),$$
$$\frac{1}{2}\rho[\rho(x_n, fz) + \rho(z, x_{n+1})]\}$$

by using (3) and letting $n \to \infty$ we obtain

$$\rho(z, fz) \leq \lambda b \max\{0,0, \rho(z, fz), \frac{1}{2}[\rho(z, fz)] \quad (4)$$

$\rho(z, fz) \leq \lambda b\rho(z, fz)$ is implies $\rho(z, fz) < \rho(z, fz)$, a contradiction, so that $z = fz$.

so z is a unique fixed point of f.

Theorem 3.2 *let $f, T: X \to X$ be a self maps on partial b-metric space (*partial b-metric Space*) such that for every x, y in X and $\alpha \in [0, \frac{1}{2b})$*

$$\rho(Tx, Ty) \leq \alpha M(x, y)$$

Where,

$$M(x, y) = \max\{\rho(fx, fy), \rho(fx, Tx), \rho(fy, Ty), \frac{1}{2}[\rho(fx, Ty) + \rho(fy, Tx)]\}. \quad (5)$$

If $TX \subseteq fX$ and one of fX or TX is a complete subspace of X. Then f and T have a coincident point. In addition f and T have a unique common fixed point u in X and $\rho(u, u) = 0$, whenever f and T are weak* compatible.

Proof. let x_0 be an arbitrary point in X. Since $TX \subseteq fX$, we can choose $x_1 \in X$ such that $fx_1 = Tx_0$, $fx_2 = Tx_1$ and $fx_3 = Tx_2$, continuing this process we have $fx_{n+1} = Tx_n, n \geq 0$

Now if $fx_n = fx_{n+1}$, for some $n \in N$, then $fx_n = fx_{n+1} = Tx_n$, T and f have a coincidence point. Assume $fx_n \neq fx_{n+1}$ for every $n \in N$, for the sake of convenience assume

$$\rho_n = \rho(fx_n, fx_{n+1}).$$

we claim that $\rho_n \leq \rho_{n-1}$.

By using (5) we have

$$\rho(fx_n, fx_{n+1}) = \rho(Tx_{n-1}, Tx_n) \leq \lambda M(x_{n-1}, x_n)$$
$$M(x_{n-1}, x_n) = \max\{\rho(fx_{n-1}, fx_n), \rho(fx_{n-1}, Tx_{n-1}, \rho(fx_n, Tx_n),$$
$$\frac{1}{2}[\rho(fx_{n-1}, Tx_n) + \rho(fx_n, Tx_{n-1})]\}$$
$$= \max\{\rho(fx_{n-1}, fx_n), \rho(fx_{n-1}, fx_n), \rho(fx_n, fx_{n+1}),$$

$$\frac{1}{2}[\rho(fx_{n-1},fx_{n+1})+\rho(fx_n,fx_n)]\}$$
$$=\max\{\rho(fx_{n-1},fx_n),\rho(fx_n,fx_{n+1}),$$
$$\frac{1}{2}[\rho(fx_{n-1},fx_{n+1})+\rho(fx_n,fx_n)]\}.$$

There is three cases.

1. If
$$M(x_{n-1},x_n)=\rho(fx_{n-1},fx_n),$$
then
$$\rho(fx_n,fx_{n+1})\leq\lambda\rho(fx_{n-1},fx_n)$$
$$<\rho(x_{n-1},x_n).$$

2. If
$$M(x_{n-1},x_n)=\rho(fx_n,fx_{n+1}),$$
then
$$\rho(fx_n,fx_{n+1})<\rho(fx_n,fx_{n+1})$$
is contraction.

3. If
$$M(x_{n-1},x_n)=\frac{1}{2}[\rho(fx_{n-1},fx_{n+1})+\rho(fx_n,fx_n)],$$
then
$$\rho(fx_n,fx_{n+1})\leq\frac{\lambda}{2}[\rho(fx_{n-1},fx_{n+1})+\rho(fx_n,fx_n)]$$

from partial b-metric triangular property we have
$$\frac{\lambda}{2}[\rho(fx_{n-1},fx_{n+1})+\rho(fx_n,fx_n)]\leq$$
$$\frac{\lambda}{2}(b[\rho(fx_{n-1},fx_n)+\rho(fx_n,fx_{n+1})-\rho(fx_n,fx_n)])$$
$$+\frac{\lambda}{2}\rho(fx_n,fx_n)$$
$$\leq\frac{\lambda b}{2}[\rho_{n-1}+\rho_n].$$
where $\lambda\in[0,\frac{1}{2b})$, then $\rho_n\leq\alpha[\rho_{n-1}+\rho_n]$, where $\alpha\in[0,\frac{1}{4})$.

Therefore $\rho_n\leq\beta\rho_{n-1}$, where $\beta=\frac{\alpha}{1-\alpha}$.

Repeating this process, we have
$$\rho_n\leq\beta\rho_0 \tag{6}$$
$\rho(fx_{n+1},fx_n)\leq\beta^n\rho(fx_1,fx_0)$ for all $n\geq0$, therefore
$$\lim_{n\to\infty}\rho_n=0$$
In view of theorem (3.1)$\{fx_n\}$ is a cauchy sequence in fX. Since fX is a complete (Pb.M.S), we have $\{fx_n\}$ is converge to some point u in X, that
$$\lim_{n\to\infty}fx_n=u$$
.

Also the subsequences $\{fx_{n(\mathrm{k})}\}$ and $\{fx_{m(\mathrm{k})}\}$are convergent to u.

There exists $z\in X$, such that $u=fz$
$$\rho(fx_n,u)=\lim_{n,m\to\infty}\rho(fx_n,fx_m)=\rho(u,u)=0 \tag{7}$$
Now, we claim $fz=Tz$ suppose, the contrary that $\rho(Tz,fz)>0$.

Then,

$$\rho(fz,Tz) \leq b[\rho(fz,Tx_{n+1}) + \rho(Tx_{n+1},Tz)] - \rho(Tx_{n+1},Tx_{n+1})$$
$$\leq b[\rho(fz,Tx_{n+1}) + \rho(Tx_{n+1},Tz)]$$

from (5) we have

$$\rho(Tx_{n+1},Tz) \leq \lambda M(x_{n+1},z)$$
$$\leq \lambda \max\{\rho(fx_{n+1},fz), \rho(fx_{n+1},Tx_{n+1}), \rho(fz,Tz),$$
$$\tfrac{1}{2}[\rho(fx_{n+1},Tz), \rho(fz,Tx_{n+1})]$$
$$\leq \lambda \max\{\rho(fx_{n+1},fz), \rho(fx_{n+1},fx_{n+2}), \rho(fz,Tz),$$
$$\tfrac{1}{2}[\rho(fx_{n+1},Tz), \rho(fz,fx_{n+2})].$$

Hence

$$\rho(fz,Tz) \leq b[\rho(fz,Tx_{n+1}) + b\lambda \max\{\rho(fx_{n+1},fz),$$
$$\rho(fx_{n+1},fx_{n+2}), \rho(fz,Tz), \tfrac{1}{2}[\rho(fx_{n+1},Tz), \rho(fz,fx_{n+2})].$$

By using (5) and letting $n \to \infty$ we obtain

$$\rho(fz,Tz) \leq b\lambda \max\{0,0,\rho(fz,Tz), \tfrac{1}{2}\rho(fz,Tz)]$$
$$\rho(fz,Tz) \leq b\lambda\rho(fz,Tz) < \rho(fz,Tz),$$

a contraction so $fz = Tz = u$ and z is coincidence point of f and T. Now to show that f and T have a common fixed point.

If f and T are weak* compatible then we have $T(fz) = f(Tz)$ whenever $fz = Tz = u$, this yields that $Tu = fu = u$

Hence u is a common fixed point of f and T

we claim that T and f have a unique common fixed point. Let u and v in X be two distinct fixed points of f and T, then

$$\rho(u,v) = \rho(Tu,Tv) \leq \alpha M(u,v)$$
$$\leq \alpha\rho(u,v) < \rho(u,v)$$

a contradiction. Hence $\rho(u,v) = 0$ and $u = v$.

Banach contraction principle in partial b-metric space.

Corollary 1 *[3] Let* (X,ρ) *be a complete* partial b-metric space *with* $b \geq 1$ *and let* $f: X \to X$ be a self map such that

$$\rho(fx,fy) \leq \alpha\rho(x,y), \tag{8}$$

for every x, y in X, where $\alpha \in [0,1)$. Then f posses a unique fixed point v in X and $\rho(v,v) = 0$.

Corollary 2 *[7] Let* (X,ρ) *be a complete* partial b-metric space *with* $b \geq 1$ *and let* $f: X \to X$ *be a self map such that*

$$\rho(fx,fy) \leq \alpha[\rho(x,fy) + \rho(y,fx)] \tag{9}$$

for every x, y in X, where $\alpha \in [0, \frac{1}{2b}]$. Then f posses a unique fixed point v in X and $\rho(v,v) = 0$.

Corollary 3 *[3] Let* (X,ρ) *be a complete* partial b-metric space *with* $b \geq 1$ *and let* $f: X \to X$ *be a self map such that*

$$\rho(fx,fy) \leq \alpha \max\{\rho(x,y), \rho(x,fx), \rho(y,fy)\} \tag{10}$$

for every x, y in X, where $\alpha \in [0,1]$. Then f posses a unique fixed point v in X and $\rho(v,v) = 0$.

with $b = 1$ in theorem (3.1) we can get corollary in partial b-metric space.

Corollary 4 *Let* (X, ρ) *be a complete (P.M.S) and let* $f: X \to X$ *be a self map such that*

$$\rho(fx, fy) \leq \alpha \max\{\rho(x, y), \rho(x, fx), \rho(y, fy), \frac{1}{2}[\rho(x, fy) + \rho(y, fx)]\}$$

for every x, y in X, where $\alpha \in [0,1]$. Then f posses a unique fixed point in X .

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حول بعض مبرهنات التقاط الصامدة في الفضاء ب - المتري الجزئي

أمل محمد هاشم و حنين عدنان بكري
قسم الرياضيات ، كلية العلوم ، جامعة البصرة ، البصرة ، العراق

المستخلص

يهدف هذا البحث الى استعراض ودراسة النقاط الصامدة المشتركه باستخدام مفهوم الفضاء الجزئي المتري وارتباطه بفئة الفضاء ب - المتري تحت الشرط الذي قدمه سيريك . النتائج التي حصلنا عليها هي تحسين وتوحيد العديد من النتائج في مبرهنات النقطه الصامده وتعميم بعض النتائج الحديثه في الفضاء ب- المتري الجزئي.