# Optimal Control Approach To Non-Linear Robust Control System With Two Uncertainties And Matching Condition

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**ABSTRACT**

In this paper, we introduce the robust control problem (RCP) of non-linear uncertain systems with the matching condition. A relationship has been developed between the robustness with perturbations (uncertainties) and the optimality condition of the current control problem. A computational algorithm for resilient control of nonlinear dynamic systems is proposed, as well as its equivalence to a specific optimal control problem (OCP.

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## 1. **Introduction**

Recently, attention has focused upon the optimization the attitude of systems. And; particularly issues related to expanding the range of missiles, The history of control systems dates back to the 18th century when James Watt used a centrifugal governor to successfully control the speed of a steamy engine. In fact, the contributions of Hazen, Minorsky, and Nyquist had a great impact on the development of control theory. The Minorsky contribution in 1922 was to play the main role in automatic control devices for transportation guidance. He explains how stability is not completely fixed by removing the system's differential conditions. Nyquist's work in 1932 focused on devising a straightforward system for computing the closed-loop systems stability by introducing the open-loop effect. While Hazen's contribution in 1934 introduced a new term in control systems called servomechanisms, which talked about the design of hand-off servomechanisms able to intently follow changing data sources [1]. The term "servomechanism" more commonly refers to a system in which the input is variable and the variable load is absent, which is an automatic tool implemented to modify a mechanism's performance through feedback of the error-sensing. This term always applies to systems that error-correction signals control and the feedback mechanical position or one of its derivatives, such as acceleration or velocity [2]. There are some application on control theory, one can see [3-8], and particularly they studied the robust control in [9]. Recently, attention has focused on optimizing the attitude of systems, particularly issues related to expanding the range of missiles, increasing the profit value of a specific project, reducing errors in estimating the position of something, reducing the energy or cost required to accomplish some final cases, or reducing the wide variety of formulations.The search for a control element that achieves the desired goal while minimizing the criterion of a specific system constitutes the basic problem of the optimization theory. To solve the optimization problem, one must first know the objective or cost function of the process that they are trying to optimize.This requires an appropriate definition of the problem in physical terms and translating the physical description into mathematical terms [10]. A controller is said to be robust if it continues to function even if its real-world system deviates from the ideal model for which it was developed. Given how complex today's cutting-edge systems have evolved, every controller must be robust.In fact, finding a model that is totally relevant to a particular system is difficult [11]. The great majority of what is associated with this exploration is expressed in Litmann (1979) [12]. The robust control stability of straight systems was focused on in state-space settings, where it demonstrated that robust control stability can be achieved due to closed criticism in light of a

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painstakingly chosen Lyupanov work that is free of vulnerability states assuming satisfying the matching condition because the most challenging aspect of a physical system to model is uncertainty [13, 14]. In order to satisfy this matching condition, the uncertainty in the system must fall within the admissible range of the principal input matrix. This strategy is extended by Barmish et al. (1985), Barmesh (1988) [15, 16], Peterson and Hollot (1986), Zak (1990), and Khargonekar (1990) [11] provided the definition of the quadratic stability, also known as stabilizability. In addition, the matching condition is essential for robust quadratic stabilisation in the event that the exponential decay must be fast and can be controlled in any way (Swei and Corless 1991; Olbort and Cieslik 1988) [17].

In [11, 18], Feng lin presented a nonlinear system with uncertainty represented as $f(y)$ with the two cases, matching and unmatching conditions, as shown in these two systems respectively;

$\dot{y} = Ay + Bu + Bf(y)$, ….. 1

and

$\dot{y} = Ay + Bu + Cf(y)$,……2

where $y \in R^n$ and $u \in R^m$. $A$ is a matrix of $R^{n\times n}$, $B$ is a matrix of $R^{n\times r}$, $C$ is a matrix of $R^{n\times q}$, $C \neq B$, and $f(y)$ is arbitrary function. Additionally in [17], he submitted a system with uncertainty in $A(p)$ as follows:

$\dot{y} = A(p)\, y + Bu$,……..3

where $p \in P$.

Motivated by the above discussions, this work tried to introduce a new system that includes two uncertainties and two cases the first with the matching condition as follows:

$\dot{y} = A(p)y + Bu + Bf(y)$,…..4

while the second with unmatching condition is given by

$\dot{y} = A(p)y + Bu + Cf(y)$ ….….5

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**2. 2 Problem formulation**

The control system is defined as follows:

$\dot{y} = A(p)y + Bu + Bf(y)$,…..6

The objective is to design a state feedback system that is capable of stabilising the system for any value of $p$ that falls within the bounds of the defined constraints and for any perturbation $f(y)$ that is acceptable. The answer to the robust problem is not yet known because the uncertainty does not fulfil a matching requirement. This requirement anticipates that the uncertainty will fall somewhere within the range of $B$. If the uncertainty satisfies the matching requirement, the RCP will typically have an answer that can be quickly uncovered by working through a mathrmLQR problem. This is the case only if the requirement has been met. The LQR problem is produced when uncertainty constraints are imposed on the cost function. The optimum control standards of the HJB condition are used as an example to demonstrate that the solution to the LQR problem is also a solution to the RCP problem (cite: Swei 1991 necessity). Where $p \in P, A(p) \in R^{n\times n}, y \in R^{n}, u \in R^{r}$, and $B \in R^{n\times r}$. These are the system coefficients, and $A(p) = A(p_0) + B\emptyset(p)$ for some $p_0 \in (a, b)$, for all $p \in (a, b)$ and $\emptyset^{\mathrm{T}}\emptyset \leq \mathrm{G}$ and $f(y)$ is an unknown nonlinear function with $f(0) = 0$. Let's assume that the function $f(y)$ is bounded in norm by a well-known function called $f_{\max}(y)$:

$\| f(y) \| \leq f_{\max}(y)$, ……. 7

The problem of finding a feedback control law $\mathrm{u} = kx$ such that at $y = 0$ the following closed-loop system $\dot{y}$

$= A(p)y + Bu + Bf(y)$,….. 8

is globally asymptomatically stable for all admissible uncertainties $p$ and f(y). In general, it is challenging to find a solution to this nonlinear RCP. Our strategy entails converting it into the OCP that is presented in the next section.

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Find a feedback control low $\mathrm{u} = \mathrm{k(x)}$ such that $x = 0$ of the closed-loop system

$\dot{x} = A(p)x + Bu + Bf(x)$.....9

is globally asymptotically stable for all admissible perturbations p and $f(x)$. This nonlinear robust control problem is difficult to solve in general. Our approach is to translate it into the following problem.

## 3 Main Results

Consider the following system:

$\dot{y} = A(p_0)y + Bu$.....10

Our goal is to find a feedback control law $\mathrm{u} = kx$ that minimises the functional cost

$\int_0^\infty (f_{\max}^2(y) + y^T y + y^T Gx + u^T u)\mathrm{dt}$,...... 11

The cost function has four terms: the cost of uncertainty is represented by terms $f_{\max}^2(\mathrm{y})$ and $y^T Gy$; the final two terms, $y^T y$ and $u^T u$, are common in optimal control. The relative importance of control and regulation can be changed by substituting $y^T y$ for $Q$ symmetric and positive semidefinite. The relationship between the OCP (3.1)-(3.2) and the problem of robust control (2.1) will be shown in the next theorem.

**Theorem 3.1** *If problem (2.1) has an optimal control solution, it is also a solution for the RCP (10 and 11).*

Proof: From the Definition

$$v(\mathrm{y}_0) = \min_u \int_0^\infty (f_{\max}^2(y) + y^T y + y^T Gx + u^T u)\mathrm{dt},$$

to be the minimum cost of bringing the system $\dot{y} = A(p)y + B(y)u$ from $y_0$ to 0.

By using Hamilton-Jacobi-Bellman equation, one can have

$$\min_u \big(f_{\max}^2(y)) + y^T Gx + y^T y + u^T u + v_y^T(y)(A(p_0)y + Bu)\big) = 0,$$

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where $v_y(y) = \partial v(y)/\partial y$ . Therefore, if u = ky (with $k^T k$ is negative definite) is the solution to the optimal control problems (3.1)-(3.2), then

$$f_{\max}^2(y) + y^T G y + y^T y + y^T k^k y + v_y^T(y)(A(p_0)y + Bky) = 0, \ldots 12$$

$$2y^T k^T + v_y^T(y)B = 0, \ldots 13$$

Now

$$\dot{v}(y) = (\partial v(y)/\partial(y)^T(dy/dt),$$

$$= v_y^T(y)(A(p_0)y + Bky + B\emptyset(p)y + Bf(y)),$$

from the Eq. (3.3) we get:

$$v_y^T(y)(A(p_0)y + BKy) = -(f_{\max}^2(y) + y^T G y + y^T y + y^T K^T K y),$$

from the Eq. (3.4) we have:

$$v_y^T B = -2y^T K^T \Rightarrow v_y^T B(\emptyset(p)y + f(y)) = 2y^T K^T \emptyset y + 2y^T K^T f(y),$$

$$\dot{v}(y) = -f_{\max}^2(y) - y^T G y - y^T y - y^T K^T K y - 2y^T \emptyset K^T - 2y^T f(y) K^T,$$

Now by adding and subtracting $y^T \emptyset^T \emptyset y$, we get

$$\begin{aligned}\dot{v} &= -f_{\max}^2(y) - y^T G y - y^T y - y^T K^T K y - 2y^T K^T f(y) - y^T K^T \emptyset y - y^T K^T \emptyset y \\ &\quad + y^T \emptyset^T \emptyset y - y^T \emptyset^T \emptyset y \\ &= -f_{\max}^2(y) - y^T y - 2y^T K^T f(y) - y^T(G - \emptyset^T \emptyset)y \\ &\quad - k^T(k - \emptyset)y - y^T(k^T + \emptyset^T)\emptyset y \\ &= -f_{\max}^2(y) - y^T y - 2y^T K^T f(y) - y^T(G - \emptyset^T \emptyset)y \\ &- y^T k^T(k + \emptyset)y - y(k + \emptyset)\emptyset^T y^T\end{aligned}$$

By adding and subtracting $y^T K^T K y$ and $f^T f$, we get

$$\begin{aligned}\dot{v} = \quad & -f_{\max}^2(y) - y^T y - y^T(G - \emptyset^T \emptyset)y - y^T(k + \emptyset)(k + \emptyset)^T y + -y^T K^T f(y) \\ & \quad -y^T K^T f(y) + y^T K^T K y - y^T K^T K y + f^T(y)f(y) - f^T(y)f(y), \\ = \quad & -f_{\max}^2(y) - y^T y - y^T(G - \emptyset^T \emptyset)y - y^T(k + \emptyset)(k + \emptyset)^T y + f^T(y)f(y) \\ & \quad +u^T u - y^T k^T(f(y) + ky) - f(y)(y^T k^T + f^T(y)) \\ = \quad & -f_{\max}^2(y) - y^T y - y^T(G - \emptyset^T \emptyset)y - y^T(k + \emptyset)(k + \emptyset)^T y + \\ & f^T(y)f(y) + y^T K^T K y - (f(y) + ky)(f(y) + ky)^T\end{aligned}$$

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From the above conditions, we get that: $(G - \emptyset^T\emptyset) \geq 0, f_{\max}^2(y) - f^T(y)f(y) \geq 0$

with $K^TK$ is negative definite we get:

$$\dot{v}(y) \leq -\big((f_{\max}^2(y) - f^T(y)f(y)) + y^Ty + y^T(G - \emptyset^T\emptyset)y$$
$$+y^T(k + \emptyset)(k + \emptyset)^Ty + (f(y) + ky)(f(y) + ky)^T\big)$$

That's mean

$$\dot{v}(\mathrm{y}) \leq -y^Ty < 0$$

As a result, the requirements of Lyapunov's theory of local stability have been met. As a consequence of this, there is a neighbourhood denoted by the expression $N = \{y: \| y(t) \| < C\}$ for some $\mathrm{C} > 0$. To the extent that if $y(t)$ enters N then $\lim_{t\to\infty} y(t) = 0$

But $y(t)$ cannot remains forever outside $N$, otherwise $y^Ty > \mathrm{C}$ For all $\mathrm{t} > 0$, therefore

$$\begin{aligned} v(y(t)) - v(y(0)) = &\int_0^t \dot{v}(y(s))ds \\ &\leq -\int_0^t \| y(s) \|^2 \, ds \\ &\leq -\int_0^t C^2 ds \\ &= -C^2\mathrm{t} \\ v(y(t)) \leq v(y(0)) - &\mathrm{C}^2\mathrm{t} \end{aligned}$$

Letting $t \to \infty$, we have $v(y(t)) \to -\infty$ which contradicts the fact that $v(y(t)) > 0$. Therefore $\lim_{t\to\infty} y(t) = 0$.

This theorem demonstrated that we can solve the OCP rather than the RCP. We have observed that all the perturbed systems are stabilised by this constant control. Our method has the advantage that solving an OCP is frequently simpler than solving a RCP. Along with the numerical techniques demonstrated in techniques demonstrated in [19-21].. It is clear that if the RCP cannot be solved, neither can the OCP.

### 3.1 Illustrated Examples

This section provided an illustrative example that confirmed the efficiency and efficacy of the proposed method.

**Example 3.1** *Assume the following control system:*

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$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1+p & p\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u+\begin{bmatrix}0\\ 1\end{bmatrix}qy_1\sin y_2,$$

where $p\in[-10,1], q\in[-1,1]$. Notice that the system is of the form $\dot{y}=A(p)y+Bu+Bf(y)$ is asymptotically stable with

$$A(p)=\begin{bmatrix}0 & 1\\ 1+\mathrm{p} & \mathrm{p}\end{bmatrix}, B=\begin{bmatrix}0\\ 1\end{bmatrix}, f(y)=qy_1\sin y_2, \ldots\ldots 14$$

$$|f|\le|y_1|=f_{\max}(y),$$

We would like to design a reliable control system with the form $u=Ky$ so that the closed-loop system is asymptotically stable for all values of $p\in,[-10,1], q\in[-1,1]$. Let's say we want to turn this problem into an OCP, so let's choose the value $p_0=0$ and investigate whether or not $(A(p_0),B)$ is controllable. The controllability matrix of $(A(p_0),B)$ is

$$C=[B \quad A(p_0)B]=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},$$

Since $C$ is of full rank, the nominal system is controllable. By rewriting the above system, we get that:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u+\begin{bmatrix}0\\ 1\end{bmatrix}[p \quad p]\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}qy_1\sin y_2,$$

Clearly, this system is equivalent to the form

$$\dot{y}=A(P_0)x+Bu+B\emptyset(p)y+Bf(y),$$

i. e. $\emptyset(p)=[p \quad p], \emptyset(p)^T\emptyset(p)=\begin{bmatrix}P^2 & P^2\\ P^2 & P^2\end{bmatrix}\le\begin{bmatrix}100 & 100\\ 100 & 100\end{bmatrix}=G$, Then, we can write the identical LQR problem such as the following nominal system

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u,$$

We must find a feedback control law $u=-ky$ that minimizes the cost function

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$$\int_0^\infty (f_{\max}^2(y) + y^T G y + y^T y + u^T u) dt$$
$$= \int_0^\infty (y_1^2 + y^T G y + y^T y + u^T u) dt$$
$$= \int_0^\infty \left( y_1^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} y_1 + y^T (G + I) y + u^T u \right) dt$$
$$= \int_0^\infty \left( y^T \begin{bmatrix} 102 & 100 \\ 100 & 101 \end{bmatrix} y + u^T u \right) dt$$

With Matlab and with $Q = \begin{bmatrix} 102 & 100 \\ 100 & 101 \end{bmatrix}$, and R = I = 1 We can solve the algebraic Riccati equation :

$$PA + A^T P + Q - PBR^{-1}B^T P = 0$$

To get P $= \begin{bmatrix} 12.6928 & 11.1489 \\ 11.1489 & 11.1040 \end{bmatrix}$ and the corresponding control is

$$u = -B^T P y = [-11.1489 - 11.1040] y$$

The figures below show the optimal control and the robust control and the solution of the system at the arbitrary values $p$ and $q$.

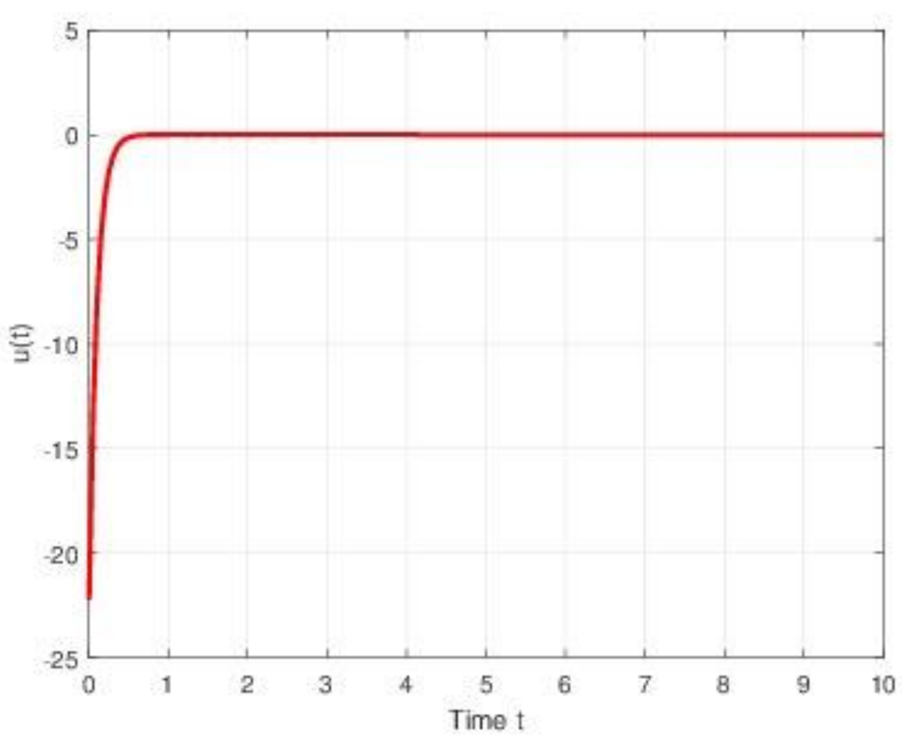


Figure 1: The Optimal Control u

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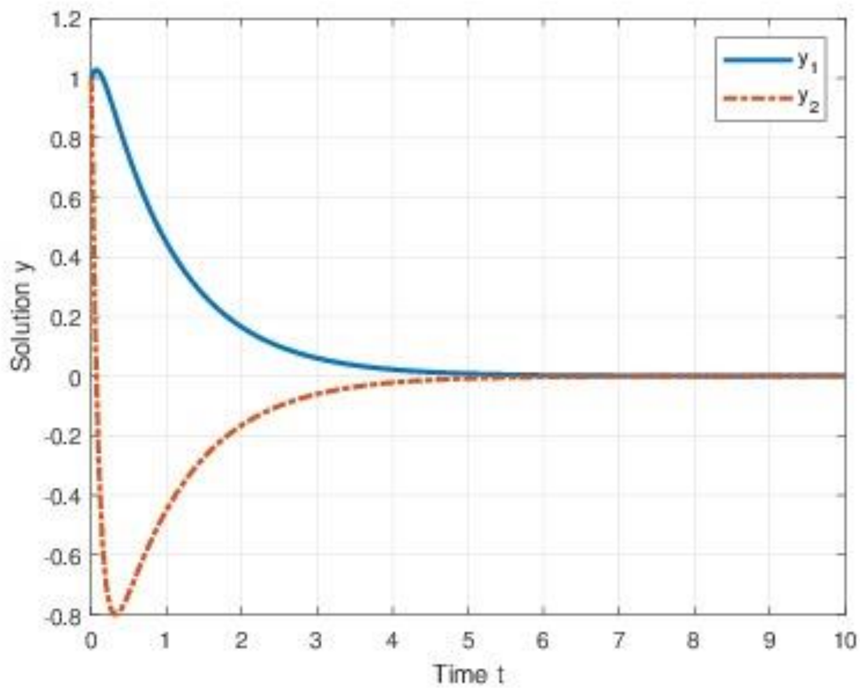


Figure 2: The optimal Solution

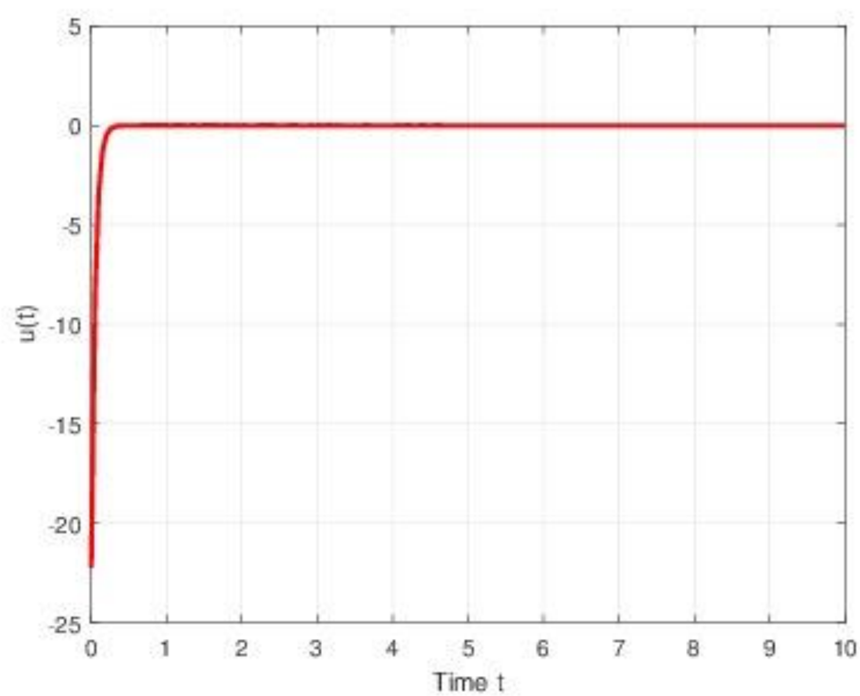


Figure 3: The Robust control u at p=-9, q=0

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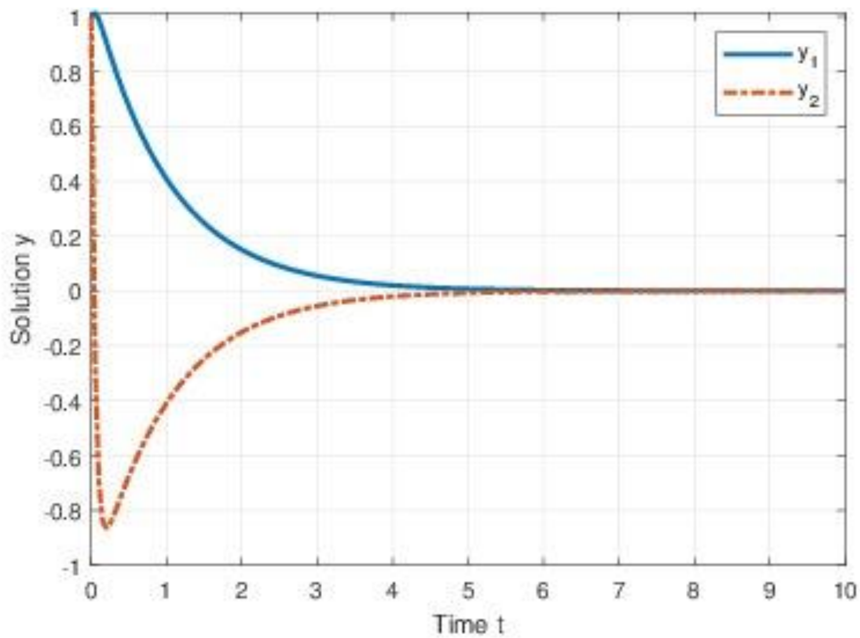


Figure 4: The Robust Solution at p=-9, q=0

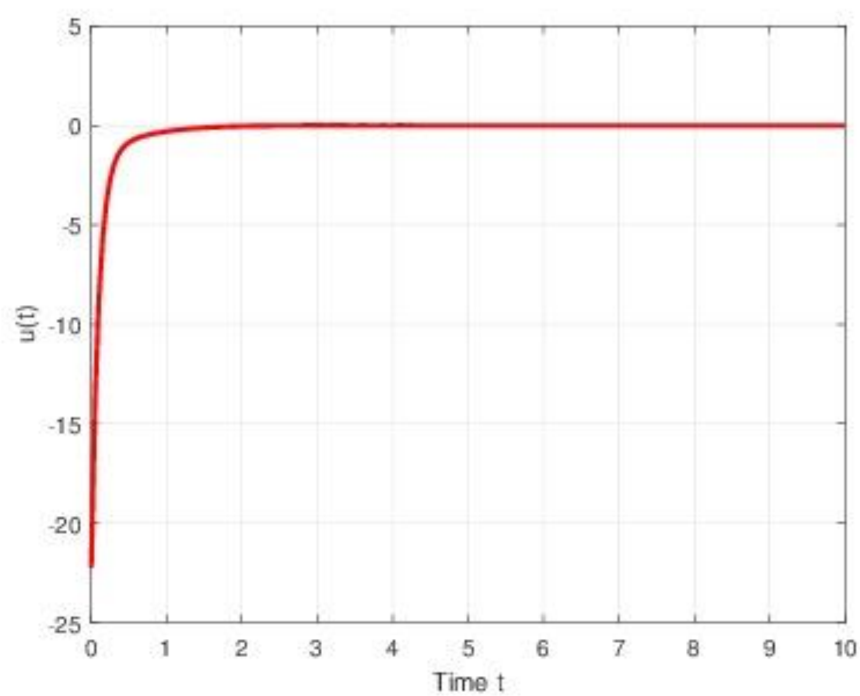


Figure 5: The Robust control u at p=1, q=-1

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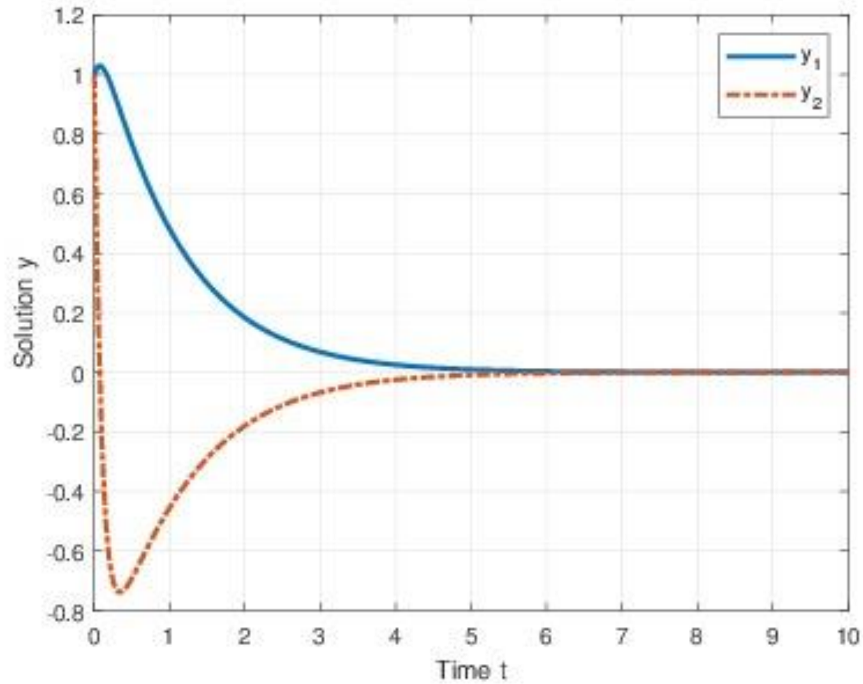


Figure 6: The Robust Solution at p=1,q=-1

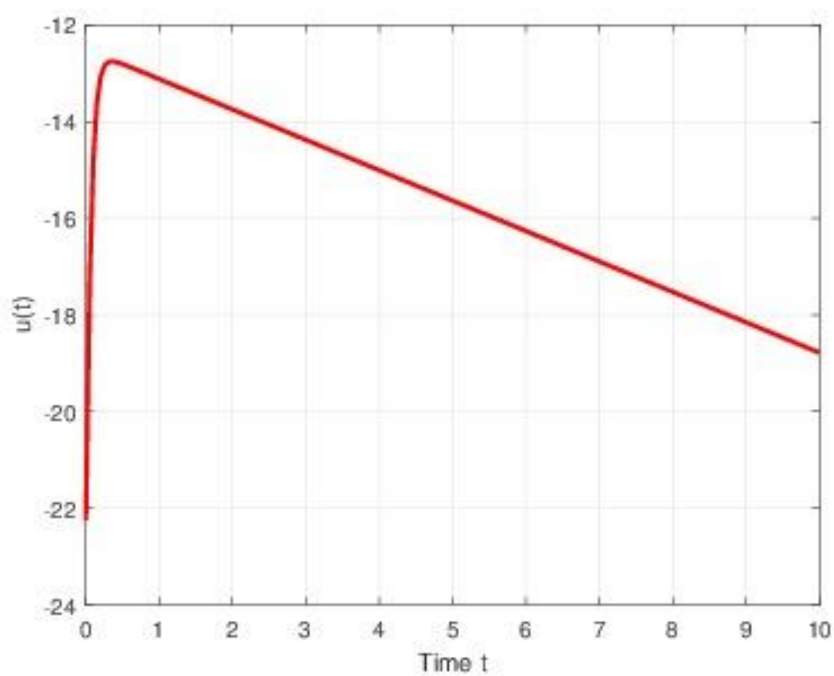


Figure 7: The Robust Control u at p=11, q=-15

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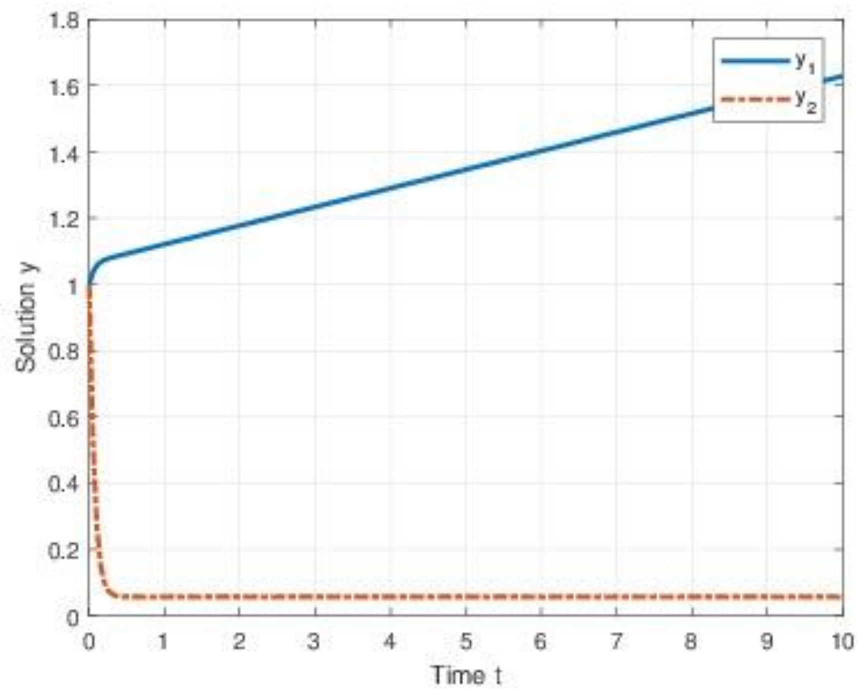


Figure 8: The Robust Solution at p=11, q=-15.

## 4 Conclusions

In this paper, the robust control in the stability of linear systems with nonlinear perturbation it is described by transforming the robust control problem with a cost function into an optimal control problem. In other words, if we can transform the original problem into an optimal control problem with a well-defined solution, ie. we can solve it indirectly. Specifically, the possibility of transforming a robust control problem into an optimal control problem in a system that has two perturbations, one in the matrix $A(p)$ and the other represented by a nonlinear function $f(x)$, has been shown. Moreover, this indirect technique has been used as one of the most useful and effective methods to solve this type of system. Assuming that the uncertainties associated with robust control are restricted to the nonlinear systems we have discussed.

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## References

[1] K. Ogata, Modern control engineering, 3th Ed., Prentice hall, 2010.

[2] H. A. Simon, On the application of servomechanism theory in the study of production control, Econometrica: J Econom. Soc., 20(1952) 247–268, https://doi.org/10.2307/1907849.

[3] A. R. Khudair, State linearization of multi-input nonlinear differential algebraic control systems, Int. J Contem. Math. Sci., 8 (2013) 499–509. http://doi:10.12988/ijcms.2013.13049.

[4] A. R. Khudair, N. K. Mahdi, Feedback linearization of single-input nonlinear differential algebraic control systems, Int. Math. Forum, 8 (2013) 167–179. http://doi:10.12988/imf.2013.13017.

[5] S. L. Khalaf, H. S. Flayyih, Stability analysis of fractional sir model related to delay in state and control variables, Bas J Sci., 39(2021)204-220, http://doi.org/10.29072/basjs.202123.

[6] S. L. Khalaf, K. K. Kassid, Approximate solution of quadratic time varying optimal control problems via differential transform method, Bas. J Sci., 38(2020) 26–47, http://doi.org/10.29072/basjs.20200103.

[7] Z. A. Lazima, S. L. Khalaf, Optimal control design of the in-vivo HIV fractional model, Iraqi J Sci., 63 (2022) 3877–3888 http://doi:10.24996/ijs.2022.63.9.20.

[8] S. L. Khalaf, Stability analysis and numerical simulation of fractional cholera model, Bas. J Sci., 40(2022) 274–288, http://doi:10.29072/basjs.20220202.

[9] G. F. Abd, R. A. Zaboon, Solution of time-varying index-2 linear differential algebraic control systems via a variational formulation technique, Iraqi Jo Sci, 62(2021) 3656–3671. https://doi.org/10.24996/ijs.2021.62.10.24.

[10] A. P. Sage, C. C. White, Optimum systems control, Prentice Hall, 1977.

[11] F. Lin, An optimal control approach to robust control design, Int. J Control, 73(2000) 177–186. https://doi.org/10.1080/002071700219722.

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[12] G. Leitmann, Guaranteed asymptotic stability for some linear systems with bounded uncertainties, J. Dyn. Sys., Meas., Control. Sep., 101(1979)212-216 https://doi.org/10.1115/1.3426427.

[13] G. Zames, Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses, IEEE Transactions on Automatic Control 26(1981) 301–320, http://doi.org/10.1109/TAC.1981.1102603.

[14] G. Mattei, Robust nonlinear control: from continuous time to sampled-data with aerospace applications., Ph.D. thesis, Paris 11 (2015).

[15] B. R. Barmish, A generalization of kharitonov's four polynomial concept for robust stability problems with linearly dependent coefficient perturbations, in: 1988 American Control Conference, IEEE, 1988,1869–1875, http://doi.org/10.23919/ACC.1988.4790031.

[16] B. R. Barmish, Necessary and sufficient conditions for quadratic stabilizability of an uncertain system, J Optim Theory Appl., 46(1985) 399–408, https://doi.org/10.1007/BF00939145.

[17] F. Lin, Robust control design: an optimal control approach, John Wiley & Sons, 2007.

[18] F. Lin, R. D. Brandt, J. Sun, Robust control of nonlinear systems: Compensating for uncertainty, Int. J Control 56 (1992) 1453–1459, http://doi.org/10.23919/ACC.1990.4791280.

[19] A. E. Bryson, Applied optimal control: optimization, estimation and control, CRC Press, 1975.

[20] D. Youla, H. Jabr, J. Bongiorno, Modern wiener-hopf design of optimal controllers–part ii: The multivariable case, IEEE Transactions on Automatic Control 21(1976) 319–338, http://doi.org/10.1109/TAC.1976.1101223.

[21] D. R. A. Zaboon, "Optimal control problem to robust nonlinear descriptor control systems with matching condition," Iraqi J Sci., 63(2022)1681-1691. https://doi.org/10.24996/ijs.2022.63.4.25.

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# طريقة التحكم الأمثل لنظام التحكم القوي غير الخطي مع اثنين من حالات عدم اليقين وبشرط المطابقة

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**المستخلص**

في هذا البحث ، قدمنا حلا لمسلة التحكم القوية (RCP) للأنظمة غير الخطية غير المؤكدة مع الشرط المطابق. تم تطوير علاقة بين المتانة مع الاضطرابات (عدم اليقين) والظروف المثلى لمشكلة التحكم الحالية. تم اقتراح خوارزمية حسابية للتحكم المرن في الأنظمة الديناميكية غير الخطية ، بالإضافة إلى معادلتها لمشكلة تحكم أمثل محددة (OCP).

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